Valuing Companies by Cash Flow Discounting: Only APV does not require iteration

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The most used methods for valuing companies by Cash Flow Discounting are equity cash flow, free cash flow, capital cash flow and APV (Adjusted Present Value). Only APV does not require iteration

All four methods, if properly applied, always give the same value. This result is logical, as all the methods analyze the same reality under the same hypotheses; they differ only in the cash flows or parameters taken as the starting point for the valuation.

Many valuations are incorrect because the authors do not iterate and, therefore, the four methods do not provide the same value.

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I thank Rafael Termes and my colleagues at IESE for their sharp questions that encouraged me to explore valuation problems.

Section 1 shows the four most used methods for valuing companies by discounted cash flows:

1) equity cash flows discounted at the required return to equity
2) free cash flow discounted at the WACC
3) capital cash flows discounted at the WACC before tax
4) APV (Adjusted Present Value)

The four methods, if properly applied, always give the same value. This result is logical, since all the methods analyze the same reality under the same hypotheses; they differ only in the cash flows taken as the starting point for the valuation.

In **section 2** the four methods are applied to an example.

The formulas used in section 2 are valid if the interest rate on the debt matches the required return to debt (Kd), that is, if the debt's market value is identical to its book value. The formulas for when this is not the case, are given in Appendix 3.

Section 3 gives a brief overview of the most significant theories on the Value of Tax Shields (VTS). **Appendix 1** contains the valuation equations according to these theories. **Appendix 3** shows how the valuation equations change if the debt's market value is not equal to its nominal value. **Appendix 2** contains a list of the abbreviations used in the chapter.

1. Four discounted cash flow methods for valuing companies

There are **four basic methods** for valuing companies by discounted cash flows:

<u>**Method 1**. Using the expected equity cash flow (ECF) and the required return to equity (Ke).</u>

Equation [1] indicates that the value of the equity (E) is the present value of the expected equity cash flows (ECF) discounted at the required return to equity (Ke).

[1] $E_0 = PV_0 [Ke_t; ECF_t]$

Equation [2] indicates that the value of the debt (D) is the present value of the expected debt cash flows (CFd) discounted at the required return to debt (Kd). ΔD_t is the increase in debt, and I_t is the interest paid by the company. $CFd_t = I_t - \Delta D_t$

[2] $D_0 = PV_0 [Kd_t; CFd_t]$

<u>Definition of FCF</u>: the free cash flow is the hypothetical equity cash flow when the company has no debt. The expression that relates the FCF with the ECF is:

[3] $FCF_t = ECF_t - \Delta D_t + I_t (1 - T)$

<u>Method 2</u>. <u>Using the free cash flow and the WACC (weighted average cost of capital).</u>

Equation [4] indicates that the value of the debt (D) plus that of the shareholders' equity (E) is the present value of the expected free cash flows (FCF) discounted at the weighted average cost of capital (WACC):

[4] $E_0 + D_0 = PV_0 [WACC_t ; FCF_t]$

<u>Definition of WACC</u>: the WACC is the rate at which the FCF must be discounted so that equation [4] gives the same result as that given by the sum of [1] and [2]. By doing so, the WACC is [5]:

[5] $WACC_t = [E_{t-1} Ke_t + D_{t-1} Kd_t (1-T)] / [E_{t-1} + D_{t-1}]$

T is the tax rate used in equation [3]. $E_{t-1} + D_{t-1}$ are **not** market values nor book values: in actual fact, E_{t-1} is the value obtained when the valuation is performed using formulae [1] or [4]. Consequently, the valuation is an iterative process: the free cash flows are discounted at the WACC to calculate the company's value (D+E) but, in order to obtain the WACC, we need to know the company's value (D+E).

<u>Method 3</u>. <u>Using the capital cash flow (CCF) and the $WACC_{BT}$ (weighted average cost of capital, before tax).</u>

The capital cash flows are the cash flows available for all holders of the company's securities, whether these be debt or shares, and are equivalent to the equity cash flow (ECF) plus the cash flow corresponding to the debt holders (CFd).

Equation [6] indicates that the value of the debt today (D) plus that of the shareholders' equity (E) is equal to the capital cash flow (CCF) discounted at the weighted average cost of capital before tax ($WACC_{BT}$).

[6] $E_0 + D_0 = PV[WACC_{BTt}; CCF_t]$

Definition of $WACC_{BT}$. [7] is the rate at which the CCF must be discounted so that equation [6] gives the same result as that given by the sum of [1] and [2].

[7] $WACC_{BT\ t} = [E_{t-1}\ Ke_t + D_{t-1}\ Kd_t]\ /\ [E_{t-1} + D_{t-1}]$

The expression that relates the CCF with the ECF and the FCF is [8]:

[8] $CCF_t = ECF_t + CFd_t = FCF_t + I_t\ T.$ $\Delta D_t = D_t - D_{t-1}$; $I_t = D_{t-1}\ Kd_t$

Method 4. Adjusted present value (APV)

Equation [9] indicates that the value of the debt (D) plus that of the shareholders' equity (E) is equal to the value of the unlevered company's shareholders' equity, Vu, plus the value of the tax shield (VTS):

[9] $E_0 + D_0 = Vu_0 + VTS_0$

We can see in **section 3** that there are several theories for calculating the VTS.

Ku is the required return to equity in the debt-free company. Vu is given by [10]:

[10] $Vu_0 = PV_0\ [Ku_t; FCF_t]$

2. An example. Valuation of the company CBA Inc.

The company CBA Inc. has the balance sheet and income statement forecasts for the next few years shown in **Table 1**. After year 3, the balance sheet and the income statement are expected to grow at an annual rate of 2%.

Table 1. Balance sheet and income statement forecasts for CBA Inc.

	0	1	2	3	4	5
WCR (working capital requirements)	400	430	515	550	561.00	572.22
Gross fixed assets	1,600	1,800	2,300	2,600	2,913.00	3,232.26
- accumulated depreciation		200	450	720	995.40	1,276.31
Net fixed assets	1,600	1,600	1,850	1,880	1,917.60	1,955.95
TOTAL ASSETS	**2,000**	**2,030**	**2,365**	**2,430**	**2,478.60**	**2,528**
Debt (N)	1,500	1,500	1,500	1,500	1,530.00	1,560.60
Equity (book value)	500	530	865	930	948.60	967.57
TOTAL LIABILITIES	**2,000**	**2,030**	**2,365**	**2,430**	**2,478.60**	**2,528**
Income statement						
Margin		420	680	740	765.00	780
Interest payments		120	120	120	120.00	122
PBT (profit before tax)		300	560	620	645.00	658
Taxes		105	196	217	225.75	230.27
PAT (profit after tax = net income)		**195**	**364**	**403**	**419.25**	**427.64**

Using the balance sheet and income statement forecasts in Table 1, we can readily obtain the cash flows given in **Table 2**. Obviously, the cash flows grow at a rate of 2% after year 4.

Table 2. Cash flow forecasts for CBA Inc.

	1	2	3	4	5
PAT (profit after tax)	**195**	**364**	**403**	**419.25**	**427.64**
+ depreciation	200	250.00	270.00	275.40	280.91
+ increase of debt	0	0.00	0.00	30.00	30.60
- increase of working capital requirements	-30	-85	-35	-11	-11.22
- investment in fixed assets	-200	-500.00	-300.00	-313.00	-319.26
ECF	**165.00**	**29.00**	**338.00**	**400.65**	**408.66**
FCF [3]	**243.00**	**107.00**	**416.00**	**448.65**	**457.62**
CFd	**120.00**	**120.00**	**120.00**	**90.00**	**91.80**
CCF [8]	**285.00**	**149.00**	**458.00**	**490.65**	**500.46**

The unlevered beta (βu) is 1. The risk-free rate is 6%. The cost of debt is 8%. The corporate tax rate is 35%. The required market risk premium[1] is 4%. $Ku = R_F + \beta u\, P_M = 6\% + 4\% = 10\%$. With these parameters, the valuation of this company's equity, using the above equations, is given in **Table 3**.

Table 3. Valuation of CBA Inc., according to Fernandez (2007). No cost of leverage

equation		0	1	2	3	4	5
	Ku	10.00%	10.00%	10.00%	10.00%	10.00%	10.00%
	Ke	10.49%	10.46%	10.42%	10.41%	10.41%	10.41%
[1]	**E = PV(Ke;ECF)**	**3,958.96**	**4,209.36**	**4,620.80**	**4,764.38**	**4,859.66**	**4,956.86**
[2]	D = PV(CFd;Kd)	1,500.00	1,500.00	1,500.00	1,500.00	1,530.00	1,560.60
[4]	E+D = PV(WACC;FCF)	5,458.96	5,709.36	6,120.80	6,264.38	6,389.66	6,517.46
[5]	WACC	9.04%	9.08%	9.14%	9.16%	9.16%	9.16%
	[4] - D = E	**3,958.96**	**4,209.36**	**4,620.80**	**4,764.38**	**4,859.66**	**4,956.86**
[6]	D+E = PV($WACC_{BT}$;CCF)	5,458.96	5,709.36	6,120.80	6,264.38	6,389.66	6,517.46
[7]	$WACC_{BT}$	9.81%	9.82%	9.83%	9.83%	9.83%	9.83%
	[6] - D = E	**3,958.96**	**4,209.36**	**4,620.80**	**4,764.38**	**4,859.66**	**4,956.86**
	VTS = PV(Ku;D T Ku)	623.61	633.47	644.32	656.25	669.38	682.76
[10]	Vu = PV(Ku;FCF)	4,835.35	5,075.89	5,476.48	5,608.12	5,720.29	5,834.69
[9]	VTS + Vu	5,458.96	5,709.36	6,120.80	6,264.37	6,389.66	6,517.46
	[9] - D = E	**3,958.96**	**4,209.36**	**4,620.80**	**4,764.37**	**4,859.66**	**4,956.86**

The required return to equity (Ke) appears in the second line of the table. It changes every year because the relation (D_t/E_t) changes every year. To calculate Ke correctly it is necessary to iterate. The required return to equity (Ke) has been calculated according to Fernandez (2007) (see section 3). Equation [1] enables the value of the equity to be obtained

[1] About the 4 different meanings of the Market Risk Premium see *Equity Premium: Historical, Expected, Required and Implied*, http://ssrn.com/abstract=933070

by discounting the equity cash flows at the required return to equity (Ke). Likewise, equation [2] enables the value of the debt to be obtained by discounting the debt cash flows at the required return to debt (Kd). The value of the debt is equal to the nominal value (book value) given in Table 1 because we have considered that the required return to debt is equal to its cost (8%).

Another way to calculate the value of the equity is using equation [4]. The present value of the free cash flows discounted at the WACC (equation [5]) gives us the value of the company, which is the value of the debt plus that of the equity. By subtracting the value of the debt from this quantity, we obtain the value of the equity. The WACC changes every year because the relation (D_t/E_t) and Ke change every year. To calculate WACC correctly it is necessary to iterate.

Another way of calculating the value of the equity is using equation [6]. The present value of the capital cash flows discounted at the $WACC_{BT}$ (equation [7]) gives us the value of the company, which is the value of the debt plus that of the equity. By subtracting the value of the debt from this quantity, we obtain the value of the equity. The $WACC_{BT}$ changes every year because the relation (D_t/E_t) and Ke change every year. To calculate $WACC_{BT}$ correctly it is necessary to iterate.

The fourth method for calculating the value of the equity is using the Adjusted Present Value, equation [9]. The value of the company is the sum of the value of the unlevered company (equation [10]) plus the present value of the value of the tax shield (VTS). As the required return to equity (Ke) has been calculated according to Fernandez (2007), we must also calculate the VTS accordingly: VTS = PV (Ku; D T Ku).

Table 3 shows that the result obtained with all ten valuation methods is the same. The value of the equity today is 3,958.96. As I have already mentioned, these valuations have been performed according to the Fernandez (2007) theory. The valuations performed using other theories about the VTS (Value of Tax Shields) are discussed in section 4.

3. An overview of the most significant papers about the value of tax shields (VTS)

There is a considerable body of literature on the discounted cash flow valuation of firms. I will now discuss the most salient papers, concentrating particularly on those that proposed different expressions for the present value of the tax savings due to the payment of interest or value of tax shields (VTS)[2].

Modigliani and Miller (1958) studied the effect of leverage on the firm's value. Their proposition 1 (1958, equation 3) states that, in the absence of taxes, the firm's value is independent of its debt, i.e.

[23] E + D = Vu, if T = 0.

E is the equity value, D is the debt value, Vu is the value of the unlevered company, and T is the tax rate.

In the presence of taxes and for the case of a perpetuity, they calculate the value of tax shields (VTS) by discounting the present value of the tax savings due to interest payments on a risk-free debt (T D R_F) at the risk-free rate (R_F). Their first proposition, with taxes, is transformed into Modigliani and Miller (1963, page 436, equation 3):

[24] E + D = Vu + PV[R_F; DT R_F] = Vu + D T

DT is the value of tax shields (VTS) for perpetuity. This result is only correct for perpetuities. As **Fernandez (2004 and 2007)** demonstrates, discounting the tax savings due to interest payments on a risk-free debt at the risk-free rate provides inconsistent results for growing companies. We have seen this in Table 13.

Myers (1974) introduced the APV (adjusted present value). According to Myers, the value of the levered firm is equal to the value of the firm with no debt (Vu) plus the present value of the tax saving due to the payment of interest (VTS). Myers proposes calculating the VTS by discounting the tax savings (D T Kd) at the cost of debt (Kd). The argument is that the risk of the tax saving arising from the use of debt is the same as the risk of the debt. Therefore, according to Myers (1974):

[25] VTS = PV [Kd; D T Kd]

Luehrman (1997) recommends valuing companies using the Adjusted Present Value and calculates the VTS in the same way as Myers. **Fernandez (2007)** shows that this theory yields consistent results only if the expected debt levels are fixed.

[2] The VTS is the present value of the tax savings due to the payment of interest, but Fernandez (2004) shows that it is also the difference between the present value of taxes paid by the unlevered firm and the present value of taxes paid by the levered firm.

Miller (1977) assumes no advantages of debt financing: *"I argue that even in a world in which interest payments are fully deductible in computing corporate income taxes, the value of the firm, in equilibrium, will still be independent of its capital structure."* According to Miller (1977), the value of the firm is independent of its capital structure, that is,

[26] VTS = 0.

According to **Miles and Ezzell (1980)**, a firm that wishes to keep a constant D/E ratio must be valued in a different manner from a firm that has a preset level of debt. For a firm with a fixed debt target [D/(D+E)], they claim that the correct rate for discounting the tax saving due to debt (Kd T D_{t-1}) is Kd for the tax saving during the first year, and Ku for the tax saving during the following years. The expression of Ke is their equation 22:

[27] Ke = Ku + D (Ku - Kd) [1 + Kd (1-T)] / [(1+Kd) E]

Arzac and Glosten (2005) and **Cooper and Nyborg (2006)** show that Miles and Ezzell (1980) (and their equation [27]) imply that the value of tax shields is[3]:

[28] VTS = PV[Ku; T D Kd] (1+Ku)/(1+Kd).

Harris and Pringle (1985) calculate the VTS by discounting the tax saving due to the debt (Kd T D) at the rate Ku. Their argument is that the interest tax shields have the same systematic risk as the firm's underlying cash flows and, therefore, should be discounted at the required return to assets (Ku). According to them:

[29] VTS = PV [Ku; D Kd T]

Harris and Pringle (1985, page 242) say "*the MM position is considered too extreme by some because it implies that interest tax shields are no more risky than the interest payments themselves. The Miller position is too extreme for some because it implies that debt cannot benefit the firm at all. Thus, if the truth about the value of tax shields lies somewhere between the MM and Miller positions, a supporter of either Harris and Pringle or Miles and Ezzell can take comfort in the fact that both produce a result for unlevered returns between those of MM and Miller. A virtue of Harris and Pringle compared to Miles and Ezzell is its simplicity and straightforward intuitive explanation.*" **Ruback (1995, 2002)** reaches equations that are identical to those of Harris-Pringle (1985). **Kaplan and Ruback (1995)** also calculate the VTS *"discounting interest tax shields at the discount rate for an all-equity firm".*

[3] **Lewellen and Emery** (1986) also claim that the most logically consistent method is Miles and Ezzell.

Tham and Vélez-Pareja (2001), following an arbitrage argument, also claim that the appropriate discount rate for the tax shield is Ku, the required return to unlevered equity.

Damodaran (1994, page 31**)** argues that if all the business risk is borne by the equity, then the equation relating the levered beta (β_L) to the asset beta (βu) is:

[30] $\beta_L = \beta u + (D/E)\ \beta u\ (1 - T)$.

Note that equation [30] is exactly equation [22] assuming that $\beta d = 0$. One interpretation of this assumption is that *"all of the firm's risk is borne by the stockholders (i.e., the beta of the debt is zero)".* However, I think that it is difficult to justify that the debt has no risk (unless the cost of debt is the risk-free rate) and that the return on the debt is uncorrelated with the return on assets of the firm. I rather interpret equation [30] as an attempt to introduce some leverage cost in the valuation: for a given risk of the assets (βu), by using equation [30] we obtain a higher β_L (and consequently a higher Ke and a lower equity value) than with equation [22]. Equation [30] appears in many finance books and is used by some consultants and investment banks.

Although Damodaran does not mention what the value of tax shields should be, his equation [30] relating the levered beta to the asset beta implies that the value of tax shields is:

[31] $VTS = PV[Ku;\ D\ T\ Ku - D\ (Kd - R_F)\ (1-T)]$

Another way of calculating the levered beta with respect to the asset beta is the following:

[32] $\beta_L = \beta u\ (1 + D/E)$.

We will call this method the **Practitioners' method**, because consultants and investment banks often use it (one of the many places where it appears is Ruback (1995, page 5)). It is obvious that according to this equation, given the same value for βu, a higher β_L (and a higher Ke and a lower equity value) is obtained than according to [22] and [30]. One should notice that equation [32] is equal to equation [30] eliminating the (1-T) term. We interpret equation [32] as an attempt to introduce still higher leverage cost in the valuation: for a given risk of the assets (βu), by using equation [32] we obtain a higher β_L (and consequently a higher Ke and a lower equity value) than with equation [30]. Equation [32] implies that the value of tax shields is:

[33] VTS = PV[Ku; D T Kd - D(Kd- R_F)]. [33] provides a VTS that is PV[Ku; D T (Ku- R_F)] lower than [31].

Inselbag and Kaufold (1997) argue that if the firm targets the dollar values of debt outstanding, the VTS is given by the Myers (1974) equation. However, if the firm targets a constant debt/value ratio, the VTS is given by the Miles and Ezzell (1980) equation.

Copeland, Koller and Murrin (2000) treat the Adjusted Present Value in their Appendix A. They only mention perpetuities and only propose two ways of calculating the VTS: Harris and Pringle (1985) and Myers (1974). They conclude "*we leave it to the reader's judgment to decide which approach best fits his or her situation*". They also claim that *"the finance literature does not provide a clear answer about which discount rate for the tax benefit of interest is theoretically correct."* It is quite interesting to note that Copeland et al. (2000, page 483) only suggest Inselbag and Kaufold (1997) as additional reading on Adjusted Present Value.

According to **Fernandez (2007),** the VTS is the present value of DTKu (*not* the interest tax shield) discounted at the unlevered cost of equity (Ku).
[34] PV[Ku; D T Ku]

With-Costs-Of-Leverage. This theory provides another way of quantifying the VTS:
[35] VTS = PV[Ku; D Ku T – D (Kd - R_F)]

One way of interpreting equation [35] is that the leverage costs are proportional to the amount of debt and to the difference between the required return on debt and the risk-free rate. [35] provides a VTS that is PV[Ku; D (Kd - R_F)] lower than [34].

The following table provides a synthesis of the 9 theories about the value of tax shields applied to level perpetuities.

Perpetuities. Value of tax shields (VTS) according to the 9 theories.

Theories	**Equation**	**VTS**
Fernandez	[34]	DT
Miles-Ezzell	[28]	TDKd(1+Ku)/[(1+Kd)Ku]
Modigliani-Miller	[24]	DT
Myers	[25]	DT
Miller	[26]	0
Harris-Pringle	[29]	T D Kd/Ku

Damodaran	[31]	$DT-[D(Kd-R_F)(1-T)]/Ku$
Practitioners	[33]	$D[R_F-Kd(1-T)]/Ku$
With-Costs-Of-Leverage	[35]	$D(KuT+R_F- Kd)/Ku$

Fernandez (2007) shows that only three of them may be correct:

- When the debt level is fixed, Modigliani-Miller or Myers apply, and the tax shields should be discounted at the required return to debt.

- If the leverage ratio is fixed at market value, then Miles-Ezzell applies.

- If the leverage ratio is fixed at book value, and the appropriate discount rate for the expected increases of debt is Ku, then Fernandez (2007) applies.

4. Valuation of the company CBA Inc. according to other theories about the VTS (Value of Tax Shields)

Tables 4 to 11 contain the most salient results of the valuation performed on the company CBA Inc. according to Damodaran (1994), Practitioners method, Harris and Pringle (1985), Myers (1974), Miles and Ezzell (1980), Miller (1977), With-cost-of-leverage theory, and Modigliani and Miller (1963).

Table 4. Valuation of CBA Inc. according to Damodaran (1994)

	0	1	2	3	4	5
VTS = PV[Ku; DTKu - D (Kd- R_F) (1-T)]	391.98	398.18	405.00	412.50	420.75	429.16
Ke	11.05%	10.98%	10.89%	10.86%	10.86%	10.86%
E	**3,727.34**	**3,974.07**	**4,381.48**	**4,520.62**	**4,611.04**	**4,703.26**
WACC	9.369%	9.397%	9.439%	9.452%	9.452%	9.452%
$WACC_{BT}$	10.172%	10.164%	10.153%	10.149%	10.149%	10.149%

Table 5. Valuation of CBA Inc. according to the Practitioners method

	0	1	2	3	4	5
VTS = PV[Ku; T D Kd - D(Kd- R_F)]	142.54	144.79	147.27	150.00	153.00	156.06
Ke	11.73%	11.61%	11.45%	11.41%	11.41%	11.41%
E	**3,477.89**	**3,720.68**	**4,123.75**	**4,258.13**	**4,343.29**	**4,430.15**
WACC	9.759%	9.770%	9.787%	9.792%	9.792%	9.792%
$WACC_{BT}$	10.603%	10.575%	10.533%	10.521%	10.521%	10.521%

Table 6. Valuation of CBA Inc. according to Harris and Pringle (1985), and Ruback (1995)

	0	1	2	3	4	5
VTS = PV[Ku; T D Kd]	498.89	506.78	515.45	525.00	535.50	546.21
Ke	10.78%	10.73%	10.67%	10.65%	10.65%	10.65%
E	**3,834.24**	**4,082.67**	**4,491.93**	**4,633.12**	**4,725.79**	**4,820.30**
WACC	9.213%	9.248%	9.299%	9.315%	9.315%	9.315%
$WACC_{BT}$ = Ku	10.000%	10.000%	10.000%	10.000%	10.000%	10.000%

Table 7. Valuation of CBA Inc. according to Myers (1974)

	0	1	2	3	4	5
VTS = PV(Kd;D Kd T)	663.92	675.03	687.04	700.00	714.00	728.28
Ke	10.42%	10.39%	10.35%	10.33%	10.33%	10.33%
E	**3,999.27**	**4,250.92**	**4,663.51**	**4,808.13**	**4,904.29**	**5,002.37**
WACC	8.995%	9.035%	9.096%	9.112%	9.112%	9.112%
$WACC_{BT}$	9.759%	9.765%	9.777%	9.778%	9.778%	9.778%

Table 8. Valuation of CBA Inc. according to Miles and Ezzell

	0	1	2	3	4	5
VTS = PV[Ku; T D Kd] (1+Ku)/(1+Kd)	508.13	516.16	525.00	534.72	545.42	556.33
Ke	10.76%	10.71%	10.65%	10.63%	10.63%	10.63%
E	**3,843.5**	**4,092.1**	**4,501.5**	**4,642.8**	**4,735.7**	**4,830.4**
WACC	9.199%	9.235%	9.287%	9.304%	9.304%	9.304%
$WACC_{BT}$	9.985%	9.986%	9.987%	9.987%	9.987%	9.987%

Table 9. Valuation of CBA Inc. according to Miller

	0	1	2	3	4	5
VTS = 0	0	0	0	0	0	0
Ke	12.16%	12.01%	11.81%	11.75%	11.75%	11.75%
E = Vu	**3,335.35**	**3,575.89**	**3,976.48**	**4,108.13**	**4,190.29**	**4,274.09**
WACC= Ku	10.000%	10.000%	10.000%	10.000%	10.000%	10.000%
$WACC_{BT}$	10.869%	10.827%	10.767%	10.749%	10.749%	10.749%

Table 10. Valuation of CBA Inc. according to the With-cost-of-leverage theory

	0	1	2	3	4	5
VTS = PV[Ku; D (KuT+ R_F- Kd)]	267.26	271.49	276.14	281.25	286.88	292.61
Ke	11.37%	11.29%	11.16%	11.13%	11.13%	11.13%
E	**3,602.61**	**3,847.38**	**4,252.61**	**4,389.38**	**4,477.16**	**4,566.71**
WACC	9.559%	9.579%	9.609%	9.618%	9.618%	9.618%
$WACC_{BT}$	10.382%	10.365%	10.339%	10.331%	10.331%	10.331%

Table 11. Valuation of CBA Inc. according to Modigliani and Miller

	0	1	2	3	4	5
VTS = PV[R_F; D R_F T]	745.40	758.62	772.64	787.50	803.25	819.31
Ke	10.26%	10.23%	10.20%	10.18%	10.18%	10.18%
E	**4,080.75**	**4,334.51**	**4,749.12**	**4,895.62**	**4,993.54**	**5,093.41**
WACC	8.901%	8.940%	9.001%	9.015%	9.015%	9.015%
$WACC_{BT}$	9.654%	9.660%	9.673%	9.672%	9.672%	9.672%

Table 12 is a compendium of the valuations of CBA Inc. performed according to the nine theories. Modigliani and Miller give the highest equity value (4,080.75) and Miller the lowest (3,335.35). Note that Modigliani and Miller and Myers yield a higher equity value than the Fernandez (2007) theory. This result is inconsistent, as discussed later.

Table 12. Valuation of CBA Inc. according to the nine theories

(Value in t = 0)	Equity value (E)	Value of tax Shield (VTS)	BETAe	Ke t=0	Ke t=4	WACC	$WACC_{BT}$
Fernandez	3,958.96	623.61	1.123	10.49%	10.41%	9.04%	9.81%
Miles & Ezzell	3,843.48	508.13	1.190	10.76%	10.63%	9.20%	9.99%
Modigliani & Miller	4,080.75	745.40	1.119	10.26%	10.18%	8.90%	9.65%

Myers	3,999.27	663.92	1.105	10.42%	10.33%	8.99%	9.76%
Miller	3,335.35	0.00	1.540	12.16%	11.75%	10.00%	10.87%
Harris & Pringle	3,834.24	498.89	1.196	10.78%	10.65%	9.21%	10.00%
Damodaran	3,727.34	391.98	1.262	11.05%	10.86%	9.37%	10.17%
Practitioners	3,477.89	142.54	1.431	11.73%	11.41%	9.76%	10.60%
With cost of leverage	3,602.61	267.26	1.344	11.37%	11.13%	9.56%	10.38%

Table 13 is the valuation of CBA Inc. if the growth after year 3 were 5.6% instead of 2%. Modigliani and Miller and Myers provide a required return to equity (Ke) lower than the required return to unlevered equity (Ku = 10%), which is an inconsistent result because it does not make any economic sense.

Table 13. Valuation of CBA Inc. according to the nine theories if growth after year 3 is 5.6% instead of 2%

(Value in t = 0)	Equity value (E)	Value of tax Shield (VTS)	BETAe	Ke t=0	Ke t=4	WACC	$WACC_{BT}$
Fernandez	6,887.37	1,027.01	1.071	10.28%	10.23%	9.37%	9.87%
Miles & Ezzell	6,697.19	836.83	1.109	10.44%	10.35%	9.48%	9.99%
Modigliani & Miller	12,556.56	6,696.20	1.039	***8.19%***	***8.21%***	7.87%	8.17%
Myers	7,357.80	1,497.44	1.000	***10.00%***	***9.95%***	9.19%	9.66%
Miller	5,860.36	0.00	1.307	11.23%	10.96%	10.00%	10.57%
Harris & Pringle	6,681.97	821.61	1.112	10.45%	10.36%	9.49%	10.00%
Damodaran	6,505.91	645.55	1.150	10.60%	10.47%	9.59%	10.11%
Practitioners	6,095.11	234.75	1.246	10.98%	10.78%	9.84%	10.39%
With cost of leverage	6,300.51	440.15	1.196	10.79%	10.62%	9.71%	10.25%

5. Conclusion

We have shown that the four most used methods for valuing companies by discounted cash flows always give the same value. This result is logical, since all the methods analyze the same reality under the same hypotheses; they differ only in the cash flows taken as the starting point for the valuation. The four methods analyzed are:

1) free cash flow discounted at the WACC;
2) equity cash flows discounted at the required return to equity;
3) capital cash flows discounted at the WACC before tax;
4) APV (Adjusted Present Value).

Only APV does not require iteration. Many valuations are incorrect because the authors do not iterate and, therefore, the four methods do not provide the same value.

We have also analysed nine different theories on the calculation of the VTS, which implies nine different theories on the relationship between the levered and the unlevered beta, and nine different theories on the relationship between the required return to equity and the required return to assets. The nine theories analyzed are: 1) Fernandez (2007), 2) Miles and Ezzell (1980), 3) Modigliani and Miller (1963), 4) Myers (1974), 5) Miller (1977), 6) Harris and Pringle (1985), 7) Damodaran (1994), 8) Practitioners method, and 9) With-cost-of-leverage. The disagreements among the various theories on the valuation of the firm arise from the calculation of the value of the tax shields (VTS). Using a simple example, I show that Modigliani and Miller (1963) and Myers (1974) provide inconsistent results. Appendix 1 contains the most important valuation equations according to these theories. Appendix 3 shows how the valuation equations change if the debt's market value is not equal to its book value.

Appendix 1
Valuation equations according to the main theories
Market value of the debt = Nominal value

	Fernandez (2007)	Damodaran (1994)
Ke	$Ke = Ku + \frac{D(1-T)}{E}(Ku - Kd)$	$Ke = Ku + \frac{D(1-T)}{E}(Ku - R_F)$
Ke - Ku	$D\frac{(Ku - Kd)(1-T)}{Vu + VTS - D}$	$D\frac{(Ku - R_F)(1-T)}{Vu + VTS - D}$
ßL	$\beta_L = \beta u + \frac{D(1-T)}{E}(\beta u - \beta d)$	$\beta_L = \beta u + \frac{D(1-T)}{E}\beta u$
WACC	$Ku\left(1 - \frac{DT}{E+D}\right)$	$Ku\left(1 - \frac{DT}{E+D}\right) + D\frac{(Kd - R_F)(1-T)}{E+D}$
WACCBT	$Ku - \frac{DT(Ku - Kd)}{E+D}$	$Ku - D\frac{T(Ku - R_F) - (Kd - R_F)}{E+D}$
VTS	PV[Ku; DTKu]	PV[Ku; DTKu - D (Kd- R_F) (1-T)]

	Harris-Pringle (1985) Ruback (1995)	Myers (1974)	Miles-Ezzell (1980)
Ke	$Ke = Ku + \frac{D}{E}(Ku - Kd)$	$Ke = Ku + \frac{Vu - E}{E}(Ku - Kd)$	$Ke = Ku + \frac{D}{E}(Ku - Kd)\left[1 - \frac{TKd}{1+Kd}\right]$
Ke - Ku	$D\frac{(Ku - Kd)}{Vu + VTS - D}$	$(D - VTS)\frac{(Ku - Kd)}{Vu + VTS - D}$	$D\frac{(Ku - Kd)}{Vu + VTS - D}\left[1 - \frac{TKd}{1+Kd}\right]$
ßL	$\beta_L = \beta u + \frac{D}{E}(\beta u - \beta d)$	$\beta_L = \beta u + \frac{Vu - E}{E}(\beta u - \beta d)$	$\beta_L = \beta u + \frac{D}{E}(\beta u - \beta d)\left[1 - \frac{TKd}{1+Kd}\right]$
WACC	$Ku - \frac{DKdT}{E+D}$	$Ku - \frac{VTS(Ku - Kd) + DKdT}{E+D}$	$Ku - \frac{DKdT}{E+D}\frac{1+Ku}{1+Kd_0}$
WACCBT	Ku	$Ku - \frac{VTS(Ku - Kd)}{E+D}$	$Ku - \frac{DKdT}{E+D}\frac{(Ku - Kd)}{(1+Kd_0)}$
VTS	PV[Ku; T D Kd]	PV[Kd; T D Kd]	PV[Ku; T D Kd] (1+Ku)/(1+Kd)

	Miller	With-cost-of-leverage
Ke	$Ke = Ku + \frac{D}{E}[Ku - Kd(1-T)]$	$Ke = Ku + \frac{D}{E}[Ku(1-T) + KdT - R_F]$
Ke-Ku	$D\frac{Ku - Kd(1-T)}{Vu + VTS - D}$	$D\frac{Ku(1-T) + KdT - R_F}{Vu + VTS - D}$
ßL	$\beta_L = \beta u + \frac{D}{E}(\beta u - \beta d) + \frac{D}{E}\frac{TKd}{P_M}$	$\beta_L = \beta u + \frac{D}{E}[\beta u(1-T) + \beta dT]$
WACC	Ku	$Ku - \frac{D(KuT - Kd + R_F)}{E+D}$
WACCBT	$Ku + \frac{DKdT}{E+D}$	$Ku - \frac{D[(Ku - Kd)T + R_F - Kd)]}{E+D}$
VTS	0	PV[Ku; D (KuT+ R_F- Kd)]

	Modigliani-Miller	Practitioners
Ke	$Ke = Ku + \frac{D}{E}[Ku - Kd(1-T) - (Ku-g)\frac{VTS}{D}]$ *	$Ke = Ku + \frac{D}{E}(Ku - R_F)$
Ke-Ku	$\frac{D[Ku - Kd(1-T)] - VTS(Ku-g)}{Vu + VTS - D}$ *	$D\frac{(Ku - R_F)}{Vu + VTS - D}$
$ß_L$	$\beta_L = \beta u + \frac{D}{E}[\beta u - \beta d + \frac{TKd}{P_M} - \frac{VTS(Ku-g)}{D\ P_M}]$ *	$\beta_L = \beta u + \frac{D}{E}\beta u$
WACC	$\frac{D\ Ku - (Ku-g)\ VTS}{(E+D)}$ *	$Ku - D\frac{R_F - Kd(1-T)}{E+D}$
$WACC_{BT}$	$\frac{DKu - (Ku-g)VTS + DTKd}{E+D}$ *	$Ku + D\frac{Kd - R_F}{E+D}$
VTS	$PV[R_F;\ T\ D\ R_F]$	$PV[Ku;\ T\ D\ Kd - D(Kd - R_F)]$

* Valid only for growing perpetuities

Equations common to all methods:

$$WACC_t = \frac{E_{t-1}Ke_t + D_{t-1}Kd_t(1-T)}{E_{t-1} + D_{t-1}} \qquad WACC_{BT\ t} = \frac{E_{t-1}Ke_t + D_{t-1}Kd_t}{E_{t-1} + D_{t-1}}$$

Relationships between cash flows:

$ECF_t = FCF_t + (D_t - D_{t-1}) - D_{t-1}Kd_t(1-T)$ $\quad CCF_t = FCF_t + D_{t-1}Kd_t T$ $\quad CCF_t = ECF_t - (D_t - D_{t-1}) + D_{t-1}Kd_t$

Appendix 2. Dictionary

βd = Beta of debt $\quad \beta_L$ = Beta of levered equity $\quad \beta u$ = Beta of unlevered equity
D = Value of debt N = Book value of the debt
E = Value of equity Ebv = Book value of equity ECF = Equity cash flow FCF = Free cash flow
EP = Economic Profit EVA = Economic value added
g = Growth rate of the constant growth case I = Interest paid
Ku = Required return to unlevered equity. Ke = Required return to levered equity
Kd = Required return to debt r = Cost of debt R_F = Risk-free rate
NOPAT = Net Operating Profit After Tax = profit after tax of the unlevered company
PAT = Profit after tax PBT = Profit before tax P_M = Required Market Premium
PV = Present value T = Corporate tax rate VTS = Value of the tax shield
Vu = Value of equity in the unlevered company
WACC = Weighted average cost of capital $WACC_{BT}$ = Weighted average cost of capital before taxes
WCR = Working capital requirements

Appendix 3
Valuation equations according to the main theories when the debt's market value (D) is not equal to its nominal or book value (N)

This appendix contains the expressions of the basic methods for valuing companies by discounted cash flows when the debt's market value (D) is not equal to its nominal value (N). If D is not equal to N, it is because the required return to debt (Kd) is different from the cost of the debt (r).

The interest paid in a period t is: $I_t = N_{t-1}\ r_t$. The increase in debt in period t is: $\Delta N_t = N_t - N_{t-1}$. Consequently, the debt cash flow in period t is: $CFd = I_t - \Delta N_t = N_{t-1}\ r_t - (N_t - N_{t-1})$.

Consequently, the value of the debt at t=0 is:

$$D_0 = \sum_{t=1}^{\infty} \frac{N_{t-1}\, r_t - (N_t - N_{t-1})}{\prod_{1}^{t}(1+Kd_t)}$$

It is easy to show that the relationship between the debt's market value (D) and its nominal value (N) is:

$D_t - D_{t-1} = N_t - N_{t-1} + D_{t-1}\, Kd_t - N_{t-1}\, r_t$ Consequently: $\Delta D_t = \Delta N_t + D_{t-1}\, Kd_t - N_{t-1}\, r_t$

The fact that the debt's market value (D) is not equal to its nominal value (N) affects several equations given in section 1. Equations [1], [4], [5], [6], [7], [9] and [10] continue to be valid, but the other equations change.

The expression of the WACC in this case is: **[5*]** $WACC = \dfrac{E\,Ke + D\,Kd - N\,r\,T}{E + D}$

The expression relating the ECF to the FCF is: **[3*]** $ECF_t = FCF_t + (N_t - N_{t-1}) - N_{t-1}\ r_t\,(1 - T)$

The expression relating the CCF to the ECF and the FCF is:

[8*] $CCF_t = ECF_t + CFd_t = ECF_t - (N_t - N_{t-1}) + N_{t-1}\, r_t = FCF_t + N_{t-1}\, r_t\, T$

	Fernandez (2007)	Damodaran (1994)	Practitioners
WACC	$Ku - \dfrac{N\,rT + DT(Ku - Kd)}{(E + D)}$	$Ku - \dfrac{N\,rT + D[T(Ku - R_F) - (Kd - R_F)]}{(E + D)}$	$Ku - \dfrac{N\,rT - D(Kd - R_F)}{(E + D)}$
VTS	PV[Ku; DTKu + T(Nr-DKd)]	PV[Ku; T N r +DT(Ku- R_F) - D(Kd- R_F)]	PV[Ku; T N r - D(Kd- R_F)]

	Harris-Pringle (1985) Ruback (1995)	Myers (1974)	Miles-Ezzell (1980)
WACC	$Ku - \dfrac{N\,rT}{(E + D)}$	$Ku - \dfrac{VTS(Ku - Kd) + N\,rT}{(E + D)}$	$Ku - \dfrac{N\,rT}{(E + D)}\dfrac{1 + Ku}{1 + Kd}$
VTS	PV[Ku; T N r]	PV[Kd; T N r]	PV[Ku_t ; N_{t-1} r_t T] (1 + Ku) / (1+ Kd)

Equations common to all the methods:

$$WACC_t = \frac{E_{t-1}\,Ke_t + D_{t-1}\,Kd_t - N_{t-1}\,r_t T}{(E_{t-1} + D_{t-1})} \qquad WACC_{BTt} = \frac{E_{t-1}\,Ke_t + D_{t-1}\,Kd_t}{(E_{t-1} + D_{t-1})}$$

$$WACC_{BTt} - WACC_t = \frac{N_{t-1}\, r_t\, T}{(E_{t-1} + D_{t-1})}$$

Relationships between the cash flows: $ECF_t = FCF_t + (N_t - N_{t-1}) - N_{t-1}\, r_t\,(1\text{-}T)$

$CCF_t = FCF_t + N_{t-1}\, r_t\, T$ $CCF_t = ECF_t - (N_t - N_{t-1}) + N_{t-1}\, r_t$

REFERENCES

Arditti, F.D. and H. Levy (1977), "The Weighted Average Cost of Capital as a Cutoff Rate: A Critical Examination of the Classical Textbook Weighted Average", *Financial Management* (Fall), pp. 24-34.

Arzac, E.R and L.R. Glosten (2005), "A Reconsideration of Tax Shield Valuation", *European Financial Management* 11/4, pp. 453-461.

Cooper, I. A. and K. G. Nyborg (2006), "The Value of Tax Shields IS Equal to the Present Value of Tax Shields", *Journal of Financial Economics* 81, pp. 215-225.

Copeland, T.E., T. Koller and J. Murrin (2000), *Valuation: Measuring and Managing the Value of Companies.* Third edition. New York: Wiley.

Damodaran, A (1994), *Damodaran on Valuation*, John Wiley and Sons, New York.

Fernandez, P. (2002), *Valuation Methods and Shareholder Value Creation,* Academic Press.

Fernandez, P. (2004), "The value of tax shields is NOT equal to the present value of tax shields", *Journal of Financial Economics*, Vol. 73/1 (July), pp. 145-165.

Fernandez, P. (2007). "A More Realistic Valuation: APV and WACC with constant book leverage ratio**",** *Journal of Applied Finance,* Fall/Winter, Vol.17 No 2, pp. 13-20.

Fernandez, P. (2013), "Equity Premium: Historical, Expected, Required and Implied" Available at SSRN: http://ssrn.com/abstract=933070

Harris, R.S. and J.J. Pringle (1985), "Risk-Adjusted Discount Rates Extensions form the Average-Risk Case", *Journal of Financial Research* (Fall), pp. 237-244.

Inselbag, I. and H. Kaufold (1997), "Two DCF Approaches for Valuing Companies under Alternative Financing Strategies (and How to Choose Between Them)", *Journal of Applied Corporate Finance* (Spring), pp. 114-122.

Kaplan, S. and R. Ruback (1995), "The Valuation of Cash Flow Forecasts: An Empirical Analysis", *Journal of Finance,* Vol 50, No 4, September.

Lewellen, W.G. and D.R. Emery (1986), "Corporate Debt Management and the Value of the Firm", *Journal of Financial Quantitative Analysis* (December), pp. 415-426.

Luehrman, T. A. (1997), "What's It Worth: A General Manager's Guide to Valuation", and "Using APV: A Better Tool for Valuing Operations", *Harvard Business Review*, (May-June), pp. 132-154.

Miles, J.A. and J.R. Ezzell (1980), "The Weighted Average Cost of Capital, Perfect Capital Markets and Project Life: A Clarification," *Journal of Financial and Quantitative Analysis* (September), pp. 719-730.

Miles, J.A. and J.R. Ezzell, (1985), "Reequationing Tax Shield Valuation: A Note", *Journal of Finance*, Vol XL, 5 (December), pp. 1485-1492.

Miller, M. H. (1977), "Debt and Taxes", *Journal of Finance* (May), pp. 261-276.

Modigliani, F. and M. Miller (1958), "The Cost of Capital, Corporation Finance and the Theory of Investment", *American Economic Review* 48, 261-297.

Modigliani, F. and M. Miller (1963), "Corporate Income Taxes and the Cost of Capital: A Correction", *American Economic Review* (June), pp. 433-443.

Myers, S.C. (1974), "Interactions of Corporate Financing and Investment Decisions - Implications for Capital Budgeting", *Journal of Finance* (March), pp. 1-25

Ruback, R. S. (1995), "A Note on Capital Cash Flow Valuation", Harvard Business School, 9-295-069.

Ruback, R. (2002), "Capital Cash Flows: A Simple Approach to Valuing Risky Cash Flows", *Financial Management* 31, pp. 85–103.

Tham, J. and I. Vélez-Pareja (2001), "The Correct Discount Rate for the Tax Shield: the N-period Case", SSRN Working Paper.