A NOTE ON THE VALUATION OF THE CORPORATE DEBT TAX SHIELD

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6/6/2023

Abstract

This note revisits the valuation of the tax shield of corporate borrowing when borrowing is not linked to enterprise value. In this case past literature has typically discounted the tax shield at the cost of borrowed capital gross of corporate tax, not net of corporate tax. Yet the latter often provides useful simplifications: the book value and the fundamental value of borrowings are often close; under the standard assumption that future borrowings are zero-net-present-value transactions, the tax shield due to future borrowings need not be explicitly forecasted and valued; levered equity value is simply equal to unlevered equity value minus the value of current borrowings.

Finally, there seems to be no obvious way to correctly use the after-tax weighted average cost of capital in valuations of levered equity.

Key words: adjusted present value, corporate valuation, corporate tax shield valuation, discount rates, residual income valuation.

JEL classification: G12; G13.

1 Introduction

The value of the corporate debt tax shield, i.e. of corporate tax savings due to the deduction of debt interest expenses from taxable corporate profit, continues to be debated, not least because of its practical relevance in valuations of levered equity. The debate has engaged academics at least since the Modigliani-Miller propositions (1958, 1963). Dempsey (2019) among others provides an effective summary of the debate evolution along the decades. The state of the debate is summarised in equation 21 of Dempsey (2019). That equation discounts corporate debt tax shields at the "appropriate" rate, where "appropriate" refers to the said unsettled debate.

This note attempts to contribute to the debate by revisiting the valuation of the debt tax shield in the general case whereby corporate borrowing is not linked to enterprise value, be it the enterprise market value or the enterprise fundamental value or the enterprise book value. In this case past literature has typically discounted debt tax shields at the cost of debt capital, and the latter is gross not net of corporate tax. This note highlights the merits of discounting debt tax shields at the cost of debt capital net of corporate tax.

A main merit is that future borrowings can often be assumed to be zero-NPV transactions, once the tax shields they generate is taken into account. If so, not only future borrowings, but also the tax shields they generate need not be explicitly forecasted and separately valued when valuing levered equity. This conclusion seems desirable because the tax shields of future borrowings are at the center of the debate mentioned above and are the most speculative component of the debt tax shield valuation.

Another merit of discounting debt tax shields at the cost of debt capital net of corporate tax is that levered equity value simply equals unlevered equity value minus the value of current outstanding debt. This result holds provided enterprise value is computed as the present value of after tax free cash flows discounted at the after-tax enterprise cost of capital. The said result holds both in the presence and in the absence of corporate taxation, both when financial leverage is negative and positive, both for cash flow based valuations and for accounting based valuations. The said result hinges on the consideration that corporate taxation affects the return on capital and the required return on capital of the enterprise and of its debt, but does not so much affect the value of the enterprise and of its debt.

Another conclusion in this note, a sombre one, is that there seems to be no obvious way to correctly use the after-tax wacc (weighted average cost of capital) in valuations of levered equity.

For the sake of clarity and to focus the analysis, in this note investors' taxation is assumed absent. The next section introduces the notation. The following sections revisit Adjusted Present Value (APV) and wacc to value levered equity in the presence of corporate taxation, using both cash flow based and accounting based valuations.

2 Definitions and notation

In this note we assume no taxation of investors who own the equity and debt issued by the enterprise. This assumption is not necessary to the conclusions, but it simplifies the analysis.

We employ the following notation:

- V_t^{NOA} is the time t fundamental value of the enterprise, i.e. of the enterprise's net operating assets;

- V_t^{NFO} is the fundamental value of net financial obligations, i.e. of the enterprise's net borrowings, which are financial obligations minus financial assets;

- $V_t^e = V_t^{NOA} - V_t^{NFO}$ is the fundamental value of the enterprise's equity;

- $r_{D,t}^*$ is the cost of capital of net financial obligations during the period $[t, t+1]$, in the absence of any corporate taxation, in short "absent taxation";

- $r_{F,t}^*$ is the cost of capital of the enterprise during $[t, t+1]$, absent taxation;

- $r_{e,t}^*$ is the cost of capital of equity during $[t, t+1]$, absent taxation;

- τ is the corporate tax rate.

The costs of capital are such that

$$r^*_{F,t} = \frac{V^e_t}{V^{NOA}_t} r^*_{e,t} + \frac{V^{NFO}_t}{V^{NOA}_t} r^*_{D,t}$$

$$r^\tau_{F,t} = \frac{V^e_t}{V^{NOA}_t} r^\tau_{e,t} + \frac{V^{NFO}_t}{V^{NOA}_t} r^\tau_{D,t}$$

$$r^\tau_{D,t} = r^*_{D,t} \cdot (1-\tau)$$

$$r^\tau_{F,t} = r^*_{F,t} \cdot (1-\tau)$$

$$r^\tau_{e,t} = r^*_{e,t} \cdot (1-\tau)\,.$$

$r^\tau_{D,t}$ is the after-tax counterpart of $r^*_{D,t}$. $r^\tau_{F,t}$ and $r^\tau_{e,t}$ are to be interpreted similarly.

We also employ the following additional notation:

- $V^{NOA}_t \in \left\{V^{NOA,U}_t, V^{NOA,L}_t\right\}$; this means that V^{NOA}_t can be $V^{NOA,U}_t$ or $V^{NOA,L}_t$;

- $V^{NFO}_t \in \left\{V^{NFO,N}_t, V^{NFO,G}_t\right\}$;

- $V^{NOA,U}_t$ is the value of the unlevered enterprise, i.e. the enterprise with no financial assets and no financial obligations;

- $V^{NOA,L}_t$ is the value of the levered enterprise, which includes the value of debt tax shields such that $V^{NOA,L}_t = V^{NOA,U}_t + V^{TS}_t$;

- V^{TS}_t is the time t value of future debt tax shields; tax shields are corporate tax savings due to the deduction of debt interest expenses from taxable corporate profit;

- $V^{NFO,G}_t$ is the value of net financial obligations gross of debt tax shields;

- $V^{NFO,N}_t$ is the value of net financial obligations net of debt tax shields, such that $V^{NFO,N}_t = V^{NFO,G}_t - V^{TS}_t$.

These definitions imply that we can compute equity value in two equivalent ways, namely

$$V_t^e = V_t^{NOA,U} - V_t^{NFO,N} = V_t^{NOA,L} - V_t^{NFO,G}.$$

Instead $V_t^e = V_t^{NOA,U} - V_t^{NFO,G}$ would not be correct because it neglects the debt tax shield, while $V_t^e = V_t^{NOA,L} - V_t^{NFO,N}$ would not be correct because it double counts the debt tax shield.

2.1 Accounting definitions

We also employ the following accounting definitions, which largely follow the widespread textbook by Penman (2013):

- NOA_t is the book value of net operating assets at time t, i.e. operating assets minus operating liabilities; even if not necessary, for simplicity we assume that deferred tax assets and deferred tax liabilities, corporate tax credits and corporate tax liabilities are entirely part of net operating assets;

- NFO_t is the book value of net financial obligations at time t, i.e. financial obligations minus financial assets;

- $NOA_t - NFO_t = CSE_t$ is the book value of common shareholders' equity at time t; preferred stock of any type is classified as a financial obligation;

- C_{t+1} is the net operating cash flow of the unlevered enterprise during the period $[t, t+1]$;

- I_{t+1} is the net cash flow due to investments and divestments in non-current operating assets, i.e. to capital expenditures, during $[t, t+1]$;

- $C_{t+1} - I_{t+1}$ is the free cash flow of the unlevered enterprise during the period $[t, t+1]$;

- OI^*_{t+1} is operating income of the unlevered enterprise gross of the corporate tax expense; although not necessary, for simplicity we assume that OI^*_{t+1} coincides with the taxable profit of the unlevered enterprise;

- OI_{t+1} is after (corporate) tax operating income of the unlevered enterprise, so that $OI_{t+1} = OI^*_{t+1} \cdot (1-\tau)$; therefore the nominal tax rate τ is also the effective tax rate;

- NFE^*_{t+1} is net financial expense during $[t, t+1]$, absent taxation; NFE^*_{t+1} may be positive or negative and is assumed to be entirely taxable or deductible for corporate tax purposes;

- $NFE^*_{t+1} = NBC_t \cdot NFO_t$, where NBC_t is net borrowing cost during $[t, t+1]$;

- $NFE_{t+1} = (1-\tau) \cdot NFE^*_{t+1}$ is after (corporate) tax net financial expense;

- after (corporate) tax earnings during $[t, t+1]$ are $\left(OI^*_{t+1} - NFE^*_{t+1}\right)(1-\tau)$ and are also equal to $(OI_{t+1} - NFE_{t+1})$.

The actual net operating cash flow of the levered enterprise during $[t, t+1]$ is $C_{t+1} + \tau \cdot NFE^*_{t+1}$, where $\tau \cdot NFE^*_{t+1}$ is the debt tax shield during $[t, t+1]$. Since we assume that all assets and liabilities due to corporate tax are part of NOA, it follows that the tax shield $\tau \cdot NFE^*_{t+1}$ coincides with corporate tax payments saved during $[t, t+1]$ thanks to the debt tax shield. Of course the tax shield may also be negative if $NFE^*_{t+1} < 0$.

Following Penman (2013) we also assume the clean surplus accounting relation

$$C_{t+1} - I_{t+1} = OI_{t+1} - (NOA_{t+1} - NOA_t).$$

2.2 Unlevered equity/enterprise value

Unlevered equity/enterprise value is $V_t^e = V_t^{NOA} = V_t^{NOA,U}$ and

$$\begin{aligned} V_t^{NOA,U} &\in \left\{ V_t^{NOA,U,\tau}, V_t^{NOA,U,*} \right\} \\ V_t^{NOA,U,\tau} &= \frac{V_{t+1}^{NOA,U,\tau} + C_{t+1} - I_{t+1}}{1 + r_{F,t}^{\tau}} \\ V_t^{NOA,U,*} &= \frac{V_{t+1}^{NOA,U,*} + C_{t+1} - I_{t+1}}{1 + r_{F,t}^{*}}. \end{aligned}$$

These two DCF valuations differ in the cost of capital/discount rate. $V_t^{NOA,U,\tau}$ accounts for the effect of corporate taxation on free cash flows and on the discount rate. $V_t^{NOA,U,*}$ accounts for the effect of corporate taxation on free cash flows, but not on the discount rate, which seems an omission. The reason is that the cost of capital measures the opportunity cost of the foregone return on an alternative investment: such return too would be subject to corporate taxation if the alternative investment was undertaken by the enterprise. For this reason in this note we measure unlevered equity/enterprise value as $V_t^e = V_t^{NOA,U,\tau}$ rather than $V_t^e = V_t^{NOA,U,*}$.

When all variables are constant over time $V_t^{NOA,U,\tau} = \frac{OI_{t+1}^* \cdot (1-\tau)}{r_F^* \cdot (1-\tau)}$. In this case corporate taxation only affects the expected return $\frac{OI_{t+1}^* \cdot (1-\tau)}{V_t^{NOA,U,\tau}}$ and the after-tax cost of capital $r_F^* \cdot (1-\tau)$, but not $V_t^{NOA,U,\tau}$. This special case highlights why enterprise valuation can be quite insensitive to enterprise taxation.

3 Valuation of levered equity as $V_t^{NOA,U,\tau} - V_t^{NFO}$

In this section we focus on valuation of levered equity as

$$V_t^e = V_t^{NOA,U,\tau} - V_t^{NFO} \tag{1}$$

and the question is how to compute V_t^{NFO}. We consider two answers to this question that are of special interest.

The first answer uses the discount rate $r_{D,t}^*$ such that:

- $V_t^{NFO} \in \left\{V_t^{NFO,G,*}, V_t^{NFO,N,*}\right\}$;

- $V_t^{NFO,G,*} = \frac{NBC_t \cdot NFO_t - (NFO_{t+1} - NFO_t) + V_{t+1}^{NFO}}{1 + r_{D,t}^*}$;

- $V_t^{NFO,N,*} = \frac{(1-\tau) \cdot NBC_t \cdot NFO_t - (NFO_{t+1} - NFO_t) + V_{t+1}^{NFO}}{1 + r_{D,t}^*}$;

- $V_t^{TS} = V_t^{TS,*} = \frac{\tau \cdot NBC_t \cdot NFO_t + V_{t+1}^{TS,*}}{1 + r_{D,t}^*}$;

- $V_t^{NFO,N,*} + V_t^{TS,*} = V_t^{NFO,G,*}$; $V_t^{NFO,N,*}$ implies that the debt tax shield is $V_t^{TS,*}$.

The above formula for V_t^{TS} assumes that the tax payments saved during $[t, t+1]$ due to the debt tax shield coincide with $\tau \cdot NBC_t \cdot NFO_t$, which seems a reasonable approximation. Below we also consider other formulae for V_t^{TS}, but they all make the same assumption.

The second and alternative answer to the question as to how to compute V_t^{NFO} uses the discount rate $r_{D,t}^{\tau}$ such that:

- $V_t^{NFO} \in \left\{V_t^{NFO,G,\tau}, V_t^{NFO,N,\tau}\right\}$;

- $V_t^{NFO,G,\tau} = \frac{NBC_t \cdot NFO_t - (NFO_{t+1} - NFO_t) + V_{t+1}^{NFO,G,\tau}}{1 + r_{D,t}^*(1-\tau)}$;

- $V_t^{NFO,N,\tau} = \frac{(1-\tau) \cdot NBC_t \cdot NFO_t - (NFO_{t+1} - NFO_t) + V_{t+1}^{NFO,N,\tau}}{1 + r_{D,t}^*(1-\tau)}$;

- $V_t^{TS} = V_t^{TS,\tau} = \frac{\tau \cdot NBC_t \cdot NFO_t + V_{t+1}^{TS,\tau}}{1 + r_{D,t}^*(1-\tau)}$;

- $V_t^{NFO,N,\tau} + V_t^{TS,\tau} = V_t^{NFO,G,\tau}$; $V_t^{NFO,N,\tau}$ implies that the value of the debt tax shield is $V_t^{TS,\tau}$.

$V_t^{NFO,N,*}$ and $V_t^{NFO,N,\tau}$ are net of the value of the debt tax shield, while $V_t^{NFO,G,*}$ and $V_t^{NFO,G,\tau}$ are "gross" of such value.

$V_t^{NFO,G,*}$ and $V_t^{NFO,N,\tau}$ can be equal to NFO_t in interesting special cases illustrated in the Appendix, unlike $V_t^{NFO,G,\tau}$ and $V_t^{NFO,N,*}$. $V_t^{NFO} = NFO_t$ is a convenient assumption and a common one in the accounting literature on equity valuation, as for example in Penman (2013). Then $V_t^{TS,*}$ is the tax shield value according to Adjusted Present Value (APV) when financial leverage is not tied to enterprise value. These reasons make $V_t^{NFO,G,*}$ and $V_t^{NFO,N,\tau}$ the more desirable candidates to take the role of V_t^{NFO} in equation 1. Note that, if $NFO_{t+1} = V_{t+1}^{NFO,N,\tau} = V_{t+1}^{NFO,G,*}$, then

$$V_t^{NFO,N,\tau} = \frac{1 + NBC_t \cdot (1-\tau)}{1 + r_{D,t+1}^* \cdot (1-\tau)} \cdot NFO_t$$
$$V_t^{NFO,G,*} = \frac{1 + NBC_t}{1 + r_{D,t}} \cdot NFO_t.$$

Thus in general $V_t^{NFO,N,\tau} \neq V_t^{NFO,G,*}$. However there are interesting special cases, illustrated in the Appendix, whereby $V_t^{NFO,G,*} = V_t^{NFO,N,\tau}$.

The remarks that follow highlight merits of valuing levered equity as

$$V_t^e = V_t^{NOA,U,\tau} - V_t^{NFO,N,\tau}. \tag{2}$$

$V_t^{NOA,U,\tau}$ in equation 2 measures firm value in a way that is independent of financial leverage: neither the discount rate nor the cash flows to be discounted depend on financial leverage. This is a convenient simplification.

The way $V_t^{NFO,N,\tau}$ in equation 2 is computed is equally applicable when financial leverage is positive or negative, which seems a desirable symmetry. Not only across different enterprises, but even for one single enterprise can financial leverage change sign from positive to negative and vice-versa.

In the case of negative leverage $V_t^{NFO,N,\tau} < 0$ because the value of the enterprise's financial assets exceeds the value of the enterprise's financial obligations. In the case of negative leverage $V_t^{NFO,N,\tau}$ is consistent with $V_t^{NOA,U,\tau}$, which is desirable, but $V_t^{NFO,G,*}$ is not. The reason is that $V_t^{NFO,N,\tau}$, unlike $V_t^{NFO,G,*}$, is computed by forecasting after-tax cash flows and discounting them at the after-tax cost of capital for such cash flows, which is the same approach as for $V_t^{NOA,U,\tau}$.

$V_t^{NFO,N,\tau}$ would typically be the value of the enterprise's current net borrowing. Then should $V_t^{NFO,N,\tau}$ also reflect the present value of future new borrowings? The answer is no if future net borrowings can be expected to be approximately zero net present value (0-NPV) transactions. They often can, but only once the tax shield due to such future borrowings is taken into account. The next example illustrates the point.

Assume a new borrowing of $\overline{NFO}_{t+1}$ at $t+1$ which entails repayment of $\left(1+\overline{NBC}_{t+1}\right)\cdot\overline{NFO}_{t+1}$ just before $t+2$, where $\overline{NBC}_{t+1} = r^*_{D,t+1}$. Then the value of the new borrowing at time $t+1$ is

$$V_{t+1}^{\overline{NFO},N,\tau} = \frac{(1-\tau)\cdot\overline{NBC}_{t+1}\cdot\overline{NFO}_{t+1} - \left(\overline{NFO}_{t+2} - \overline{NFO}_{t+1}\right) + V_{t+2}^{\overline{NFO},N,\tau}}{1 + r^*_{D,t+1}\,(1-\tau)} = \overline{NFO}_{t+1}$$

because $V_{t+2}^{\overline{NFO},N,\tau} = 0 = \overline{NFO}_{t+2}$ since the borrowing is already repaid at time $t+2$. It follows that the present value at time $t+1$ of the new borrowing is

$\overline{NFO}_{t+1} - V_{t+1}^{\overline{NFO},N,\tau} = 0$. **Thus future borrowings can often be assumed to be 0-NPV transactions once the tax shield due to them is taken into account. If so, future borrowings and the tax shields they generate do not affect $V_t^{NFO,N,\tau}$ and become irrelevant to equity valuation**. Then the issue of the rate at which to discount the tax shield due to future new borrowings, which has beset the literature for decades as mentioned in the introduction, becomes irrelevant. This conclusion does not require that $V_t^{NFO,N,\tau}$ be equal to NFO_t, even when $V_t^{NFO,N,\tau}$ only reflects current net borrowing and no future new borrowing. In fact $V_t^{NFO,N,\tau}$ and NFO_t may well differ, especially when current debt is at a fixed rate and the yield curve changes.

Assuming that future borrowings are 0-NPV transactions is consistent with reformulating financial statements for valuation purposes as taught in textbooks, e.g. Penman (2013). Reformulation for valuation purposes follows the seminal work of Feltham and Ohlson (1995) and hinges on the idea that the firm's investing and operating activities can be expected to generate or destroy value, while the financing activities of non-bank firms can often be expected not to. In fact it is for this reason that textbooks often approximate the fundamental value of net financial obligations with the book value of the same. Similarly Jing and Ohlson (2000) conclude that equity value "can be expressed in terms of financial assets (liabilities) with a coefficient of 1" plus other terms that measure the value of the firm's operations.

Assuming that future borrowings are 0-NPV transactions is also consistent with assuming that future equity transactions between the enterprise and its shareholders are 0-NPV transactions, which is a common assumption in equity valuations or enterprise valuations. Again Penman (2013) provides a textbook treatment of this assumption. One reason for this common assumption is that in an efficient market it is hardly possible to forecast gains and losses from such equity transactions. For a similar reason we can assume that also future new borrowings are 0-NPV transactions.

The just stated arguments imply that assuming that future borrowings are 0-NPV transactions is quite aligned with contemporary accounting theory. However there may be cases whereby future new borrowings are not 0-NPV transactions. In these cases $V_t^{NFO,N,\tau}$ should account for gains and losses from non-0-NPV future borrowings and future repayments. Since these gain and losses are conditional on the enterprise surviving, their present value should reflect the enterprise's credit risk. Therefore their present value should be computed using the the cost of capital r_D^τ. Instead a different discount rate may be needed when the size of non-0-NPV future net borrowings is tied to the enterprise value, but it is not clear how frequent and relevant these borrowings may be. Furthermore tying future net borrowings to enterprise value seems more questionable when future new borrowings are expected to be negative. Anyway insofar as future borrowings and repayments are 0-NPV transactions, they are irrelevant to compute $V_t^{NFO,N,\tau}$ and therefore to equity valuation.

For the reasons just given equation 2 appears a convincing way to value levered equity. On the other hand, textbook equity valuation based on discounting cash flows is

$$V_t^e = V_t^{NOA,L,*} - V_t^{NFO,G,*}$$

$$V_t^{NOA,L,*} = V_t^{NOA,U,*} + V_t^{TS,*}$$

and differs from equation 2 in two aspects:

1) the discount rates are "before tax", i.e. $r_{F,t}^*$ and $r_{D,t}^*$, not "after tax", i.e. $r_{F,t}^\tau$ and $r_{D,t}^\tau$ as in equation 2;

2) the tax shield value V_t^{TS} is added to $V_t^{NOA,U}$ to give $V_t^{NOA,L}$; this is the way Adjusted Present Value (APV) is often presented; $V_t^{NOA,L}$ is also often computed by discounting free cash flows at the after-tax wacc under the assumption that $V_t^{NFO,G} / \left(V_t^{NOA,L} - V_t^{NFO,G}\right)$ is constant over time. Further comments on the use of after-tax wacc are left to section 4.1 below.

4 Valuation of levered equity as $V_t^{NOA,L} - V_t^{NFO}$

The previous section focused on valuations of levered equity as $V_t^{NOA,U,\tau} - V_t^{NFO}$. Instead this section focuses on valuations of levered equity whereby $V_t^e = V_t^{NOA,L} - V_t^{NFO}$ where $V_t^{NOA,L}$ denotes the value of the levered enterprise such that

$$V_t^{NOA,L} \in \left\{V_t^{NOA,L,*}, V_t^{NOA,L,\tau}\right\}$$

$$V_t^{NOA,L,*} = V_t^{NOA,U,*} + V_t^{TS,*}$$

$$V_t^{NOA,L,\tau} = V_t^{NOA,U,\tau} + V_t^{TS,\tau}.$$

Again the question is how to compute $V_t^{NOA,L}$ and V_t^{NFO}.

Valuing equity as $V_t^{NOA,L} - V_t^{NFO,G}$ avoids double counting the value of the debt tax shield, while $V_t^e = V_t^{NOA,L} - V_t^{NFO,N}$ does double count. Then, the opportunity costs of capital in $V_t^{NOA,L,\tau} - V_t^{NFO,G,\tau}$ do take corporate taxation into account, while the opportunity costs of capital in $V_t^{NOA,L,*} - V_t^{NFO,G,*}$ ignore corporate taxation, which seems an omission. $V_t^{NOA,L,\tau} - V_t^{NFO,G,\tau}$ is just a re-writing of equity value in equation 2. These considerations suggest that valuing levered equity as $V_t^{NOA,L} - V_t^{NFO}$ does not seem preferable to equation 2 and at best is just the same as equation 2.

The valuations of levered equity seen so far do not use after-tax wacc. An Appendix enumerates the valuations of levered equity that do not use after-tax wacc. Next we turn to valuations of levered equity that do use after-tax wacc.

4.1 Valuations that use after-tax wacc

There seem to be two main seemingly internally consistent ways of using after-tax wacc to compute the value of the levered enterprise $V_t^{NOA,L,r_{wacc}}$. The first way starts from the unlevered enterprise value $V_t^{NOA,U,*}$, which uses the discount rate $r_{F,t}^*$. The second way start from the unlevered enterprise value $V_t^{NOA,U,\tau}$, which uses the dicount rate $r_{F,t}^\tau$. We summarise the said two ways to compute the value of the levered enterprise as follows

$$V_t^{NOA,L,r_{wacc}} = \left\{ V_t^{NOA,L,r_{wacc}^*}, V_t^{NOA,L,r_{wacc}^\tau} \right\}$$

$$V_t^{NOA,L,r^*_{wacc}} = \frac{C_{t+1} - I_{t+1} + V_{t+1}^{NOA,L,r^*_{wacc}}}{1 + r^*_{wacc,t}} = \frac{C_{t+1} - I_{t+1} + V_{t+1}^{NOA,L,r^*_{wacc}} + \tau \cdot r^*_{D,t} \cdot V_t^{NFO,G,*}}{1 + r^*_{F,t}}$$

$$V_t^{NOA,L,r^*_{wacc}} = V_t^{NOA,U,*} + V_t^{TS,*,wacc}$$

$$V_t^{TS,*,wacc} = \frac{\tau \cdot r^*_{D,t} \cdot V_t^{NFO,G,*} + V_{t+1}^{TS,*,wacc}}{1 + r^*_{F,t}}$$

$$r^*_{wacc,t} = r^*_{F,t} - \frac{V_t^{NFO,G,*} r^*_{D,t} \tau}{V_t^{NOA,L,r^*_{wacc}}} = \frac{V_t^e r^*_{e,t}}{V_t^{NOA,L,r^*_{wacc}}} + \frac{V_t^{NFO,G,*} r^*_{D,t} (1-\tau)}{V_t^{NOA,L,r^*_{wacc}}}$$

$$V_t^{NOA,L,r^\tau_{wacc}} = \frac{C_{t+1} - I_{t+1} + V_{t+1}^{NOA,L,r^\tau_{wacc}}}{1 + r^\tau_{wacc,t}} = \frac{C_{t+1} - I_{t+1} + V_{t+1}^{NOA,L,r^\tau_{wacc}} + \tau \cdot r^*_{D,t} \cdot V_t^{NFO,G,\tau}}{1 + r^\tau_{F,t}}$$

$$V_t^{NOA,L,r^\tau_{wacc}} = V_t^{NOA,U,\tau} + V_t^{TS,\tau,wacc}$$

$$V_t^{TS,\tau,wacc} = \frac{\tau \cdot r^*_{D,t} \cdot V_t^{NFO,G,\tau} + V_{t+1}^{TS,\tau,wacc}}{1 + r^\tau_{F,t}}$$

$$r^\tau_{wacc,t} = r^\tau_{F,t} - \frac{V_t^{NFO,G,\tau} r^*_{D,t} \tau}{V_t^{NOA,L,r^\tau_{wacc}}} = \frac{V_t^e r^\tau_{e,t}}{V_t^{NOA,L,r^\tau_{wacc}}} + \frac{V_t^{NFO,G,\tau} \left(r^\tau_{D,t} - r^*_{D,t}\tau\right)}{V_t^{NOA,L,r^\tau_{wacc}}}.$$

We next turn to the calculations of equity value that make use of $V_t^{NOA,L,r_{wacc}}$.

Note that when $V_t^{NFO} = V_t^{NFO,N}$ equity value as per $V_t^{NOA,L,r_{wacc}} - V_t^{NFO,N}$ implies a double count of the debt tax shield, while computing equity value as per $V_t^{NOA,L,r_{wacc}} - V_t^{NFO,G}$ avoids this double count.

However even computing equity value as

$$V_t^{NOA,L,r_{wacc}} - V_t^{NFO,G}$$

with $r_{wacc} \in \left\{r^*_{wacc,t}, r^\tau_{wacc,t}\right\}$ and $V_t^{TS,wacc} \in \left\{V_t^{TS,*,wacc}, V_t^{TS,\tau,wacc}\right\}$ poses two problems, which we now review.

The first problem is that $V_t^{TS,wacc}$ employs the discount rate $r^*_{F,t}$ or $r^\tau_{F,t}$. These rates seem at odds with the fact that the amount of net borrowing at time t is known with certainty at time t. This argument echoes a similar one by Miles and Ezzell (1980).

The second problem is the issue of how to compute $V_t^{NFO,G}$:

- when $V_t^{NFO} = V_t^{NFO,G,*}$ equity value as per $V_{t+1}^{NOA,L,r^*_{wacc}} - V_t^{NFO,G,*}$ implies that the discount rates $r^*_{F,t}, r^*_{D,t}$ overlook the effect of corporate taxation on the opportunity costs of capital, which seems an omission;

- when $V_t^{NFO} = V_t^{NFO,G,\tau}$ equity value as per $V_t^{NOA,L,r^\tau_{wacc}} - V_t^{NFO,G,\tau}$ does take into account the effect of corporate taxation on the opportunity costs of capital, but assumes that the tax shield is $V_t^{NFO,G,\tau} r^*_{D,t}\tau$ and that $NBC_t \cdot NFO_t = r^*_{D,t} V_t^{NFO,G,\tau}$; the latter is problematic because assuming $NFO_t = V_t^{NFO,G,\tau}$ seems much less defensible than assuming $NFO_t = V_t^{NFO,N,\tau}$; the fundamental value of net borrowing can often be close to its book value when it is computed by discounting after-tax cash flows at the after-tax cost of borrowed capital, as in $V_t^{NFO,N,\tau}$, not when it is computed by discounting before-tax cash flows at the after-tax cost of borrowed capital, as in $V_t^{NFO,G,\tau}$.

These two problems, and in particular the second one, discourage equity valuations as per $V_t^{NOA,L,r_{wacc}} - V_t^{NFO,G}$. Then, since also $V_t^{NOA,L,r_{wacc}} - V_t^{NFO,N}$ seems wrong to compute equity value, the sombre conclusion of this section is that there seems to be no obvious right way to compute equity value as the difference between $V_t^{NOA,L,r_{wacc}}$ and the value of net financial obligations. In other words, there seems to be no obvious way to use the ubiquitously taught after-tax wacc to value levered equity. This enhances the appeal of equity valuation as per equation 2.

5 Implications for accounting based valuations

The above analysis has considered valuations of V_t^{NOA} and V_t^{NFO} that discount cash flows. However accounting based valuations, such as Residual Operating Income Valuation (ROIV) and the Abnormal Operating Income Growth (AOIG) valuation of Ohlson and Gao (2006), are equivalent to valuations that compute V_t^{NOA} by discounting free cash flows. Thus to compute $V_t^{NOA,U,\tau}$ we can use ROIV or AOIG valuation and the discount rate $r_{F,t}^{\tau}$. Then we can compute $V_t^{NFO,N,\tau}$ using residual net financial expense valuation and the discount rate $r_{D,t}^{\tau}$. To stress this point we assume

$$\begin{aligned}
V_t^E &= \frac{d_{t+1} + V_{t+1}^E}{1 + r_{E,t}^{\tau}} \\
V_t^E &= V_t^{NOA,U,\tau} - V_t^{NFO,N,\tau} \\
r_{E,t}^{\tau} &= r_{F,t}^{\tau} + \frac{V_t^{NFO,N,\tau}}{V_t^{NOA,U,\tau} - V_t^{NFO,N,\tau}} \left(r_{F,t}^{\tau} - r_{D,t}^{\tau} \right)
\end{aligned}$$

and the clean surplus relation

$$CSE_{t+1} = CSE_t + (OI_{t+1} - NFE_{t+1}) - d_{t+1}$$

where:

- CSE_t is book value of common shareholders' equity at time t, such that $CSE_t = NOA_t - NFO_t$;

- d_{t+1} is the net payment, mainly dividends, between the enterprise and its equity holders in their capacity as equity holders during $[t, t+1]$.

This assumption implies

$$V_t^E = \frac{OI_{t+1} + NOA_t + V_{t+1}^{NOA,U,\tau} - NOA_{t+1}}{1 + r_{E,t}^{\tau}} - \frac{NFE_{t+1} + NFO_t + V_{t+1}^{NFO,N,\tau} - NFO_{t+1}}{1 + r_{E,t}^{\tau}}$$

which in turn implies

$$\begin{aligned} V_t^E &= CSE_t + \frac{OI_{t+1} - NFE_{t+1} - r_{E,t}^{\tau} \cdot CSE_t + V_{t+1}^E - CSE_{t+1}}{1 + r_{E,t}^{\tau}} \\ V_t^{NOA,U,\tau} &= NOA_t + \frac{OI_{t+1} - r_{F,t}^{\tau} \cdot NOA_t + V_{t+1}^{NOA,U,\tau} - NOA_{t+1}}{1 + r_{F,t}^{\tau}} \\ V_t^{NFO,N,\tau} &= NFO_t + \frac{NFE_{t+1} - r_{D,t}^{\tau} \cdot NFO_t + V_{t+1}^{NFO,N,\tau} - NFO_{t+1}}{1 + r_{D,t}^{\tau}}. \end{aligned} \tag{3}$$

This result is entirely consistent with equation 2. The equation for V_t^E is Residual Income Valuation (RIV). The equation for $V_t^{NOA,U,\tau}$ is ROIV. The equation for $V_t^{NFO,N,\tau}$ is residual net financial expense valuation. Accounting based valuation textbooks, for example Penman (2007), already assume $V_t^{NFO} = V_t^{NFO,N,\tau}$ and already use the discount rate $r_{D,t}^{\tau}$ to compute $V_t^{NFO,N,\tau}$ through residual financial expense valuation. However some of these textbooks also implicitly compute equity value as $V_t^e = V_t^{NOA,L,r_{wacc}} - V_t^{NFO,N,\tau}$ where

$$\begin{aligned} V_t^{NOA,L,r_{wacc}} &= NOA_t + \frac{OI_{t+1} - r_{wacc,t} \cdot NOA_t + V_t^{NOA,L,r_{wacc}} - NOA_{t+1}}{1 + r_{wacc,t}} \\ r_{wacc} &\in \left\{ r_{wacc,t}^*, r_{wacc,t}^{\tau} \right\} \end{aligned}$$

and this poses the problem already mentioned in section 4.1, namely double counting of the debt tax shield. Instead equation 2 coupled with equation 3 does not pose this problem.

Equation 3 computes $V_t^{NOA,U,\tau}$ through ROIV, but we can also compute

$V_t^{NOA,U,\tau}$ through AOIG valuation when $r_{F,t}^{\tau} = r_F^{\tau}$ for all t, i.e.

$$V_t^{NOA,U,\tau} = \frac{1}{r_F^{\tau}} \cdot \left[OI_{t+1} + \frac{AOIG_{t+2}}{1 + r_F^{\tau}} + \frac{AOIG_{t+3}}{(1 + r_F^{\tau})^2} + \frac{AOIG_{t+4}}{(1 + r_F^{\tau})^3} + .. \right]$$
$$AOIG_{t+i+1} = OI_{t+i+1} + r_F^{\tau} \cdot (C_{t+i} - I_{t+i}) - (1 + r_F^{\tau}) \cdot OI_{t+i}, \ i \geq 1.$$

Thus equation 2 is valid even when we use accounting based valuations, such as ROIV or AOIG valuation, provided we use the discount rate $r_{F,t}^{\tau}$.

6 Conclusion

This note has revisited the valuation of levered equity and of the debt tax shield. A simple way to value levered equity appears to deserve more attention. It consists in valuing unlevered equity by discounting after tax free cash flows at the enterprise after tax cost of capital, and then in subtracting the value of current outstanding net debt, computed as its after tax cash flows discounted at the after tax cost of net debt. This result holds in the presence or in the absence of corporate taxation, when financial leverage is negative or positive, for cash flow based valuations or for accounting based valuations. Past research has not highlighted the shortcomings of this valuation of levered equity, which is left to future research.

Finally, there seems to be no obvious way to correctly use the after-tax weighted average cost of capital in valuations of levered equity.

A Appendix

A.1 When $V_t^{NFO,G,*} = V_t^{NFO,N,\tau} = NFO_t$

Consider the statements:

1) $V_{t+1}^{NFO,N,\tau} = \frac{(1-\tau)\cdot NBC_{t+1}\cdot NFO_{t+1} - (NFO_{t+2} - NFO_{t+1}) + V_{t+2}^{NFO,N,\tau}}{1+r^*_{D,t+1}(1-\tau)}$;

1a) $NBC_t = r^*_{D,t}$;

1b) $V_t^{NFO,N,\tau} = NFO_t$;

1c) $V_{t+1}^{NFO,N,\tau} = NFO_{t+1}$.

This note argues that 1) implies the most desirable version of APV, when future leverage does not depend on enterprise value.

$V_t^{NFO,N,\tau}$ reflects the "after-tax" discount rate $r^*_{D,t}\cdot(1-\tau)$, which reflects the opportunity cost of borrowing. The opportunity cost of borrowing accounts for the foregone corporate tax shield from an alternative borrowing opportunity, much like the opportunity cost of capital is "after-tax" to reflect the taxation of the foregone returns on capital that could be invested in an alternative opportunity.

1) and any two of the three statements 1a), 1b), 1c) imply the third of these three statements.

Consider the statements:

2) $V_t^{NFO,G,*} = \frac{NBC_t\cdot NFO_t - (NFO_{t+1} - NFO_t) + V_{t+1}^{NFO,G,*}}{1+r^*_{D,t}}$;

2a) $NBC_t = r^*_{D,t}$;

2b) $V_t^{NFO,G*} = NFO_t$;

2c) $V_{t+1}^{NFO,G*} = NFO_{t+1}$.

2) and any two of the three statements 2a), 2b), 2c) imply the third of these three statements.

Thus, when $NBC_t = r^*_{D,t}$ it is usual to also assume $V_t^{NFO,G,*} = V_t^{NFO,N,\tau} = NFO_t$.

A.2 When $V_t^{NFO,N,\tau} = V_{t+1}^{NFO,G,*}$ and debt value is constant

Now we drop assumptions 1a) and 2a) so that $NBC_t \neq r_{D,t}$. When $\frac{NBC_t}{r_{D,t}} \cdot NFO_t$ is constant over time

$$\begin{aligned} V_t^{NFO,N,\tau} &= \frac{NBC_t \cdot (1-\tau)}{r_{D,t} \cdot (1-\tau)} \cdot NFO_t = \frac{NBC_t}{r_{D,t}} \cdot NFO_t \\ &= V_t^{NFO,G,*} = V_{t+1}^{NFO,N,\tau} = V_{t+1}^{NFO,G,*} \end{aligned}$$

and we can assume $V_t^{NFO,N,\tau} = V_t^{NFO,G,*} \neq NFO_t$.

A.3 Complete enumeration of valuations of levered equity

This Appendix provides an enumeration of the valuations in this note that do not make use of after-tax wacc. Then there are 4 ways to compute V_t^{NOA} and four ways to compute V_t^{NFO}.

The enterprise's fundamental value can be:

- $V_t^{NOA} \in \left\{ V_t^{NOA,U}, V_t^{NOA,L} \right\}$,
- $V_t^{NOA,U} \in \left\{ V_t^{NOA,U,*}, V_t^{NOA,U,\tau} \right\}$,
- $V_t^{NOA,L} \in \left\{ V_t^{NOA,L,*}, V_t^{NOA,L,\tau} \right\}$.

The enterprise's net borrowing fundamental value can be:

- $V_t^{NFO} \in \left\{V_t^{NFO,G}, V_t^{NFO,N}\right\}$,
- $V_t^{NFO,G} \in \left\{V_t^{NFO,G,*}, V_t^{NFO,G,\tau}\right\}$,
- $V_t^{NFO,N} \in \left\{V_t^{NFO,N,*}, V_t^{NFO,N,\tau}\right\}$.

The following matrix contains the ensuing 16 ways to compute the value of levered equity V_t^e

$$\begin{pmatrix} V_t^{NOA,U,*} & V_t^{NOA,U,\tau} \\ V_t^{NOA,L,*} & V_t^{NOA,L,\tau} \end{pmatrix} \otimes \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \otimes \begin{pmatrix} V_t^{NFO,G,*} & V_t^{NFO,G,\tau} \\ V_t^{NFO,N,*} & V_t^{NFO,N,\tau} \end{pmatrix}$$

where $\otimes$ is the Kroneker product. This same matrix can be re-written as a block matrix, i.e.

$$\begin{pmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{pmatrix}$$

and the four blocks are

$$\begin{aligned}
\mathbf{M}_{11} &= V_t^{NOA,U,*} - \begin{pmatrix} V_t^{NFO,G,*} & V_t^{NFO,G,\tau} \\ V_t^{NFO,N,*} & V_t^{NFO,N,\tau} \end{pmatrix} \\
\mathbf{M}_{12} &= V_t^{NOA,U,\tau} - \begin{pmatrix} V_t^{NFO,G,*} & V_t^{NFO,G,\tau} \\ V_t^{NFO,N,*} & V_t^{NFO,N,\tau} \end{pmatrix} \\
\mathbf{M}_{21} &= V_t^{NOA,L,*} - \begin{pmatrix} V_t^{NFO,G,*} & V_t^{NFO,G,\tau} \\ V_t^{NFO,N,*} & V_t^{NFO,N,\tau} \end{pmatrix} \\
\mathbf{M}_{22} &= V_t^{NOA,L,\tau} - \begin{pmatrix} V_t^{NFO,G,*} & V_t^{NFO,G,\tau} \\ V_t^{NFO,N,*} & V_t^{NFO,N,\tau} \end{pmatrix}.
\end{aligned}$$

We briefly comment on each of these 16 ways to compute V_t^e:

- $V_t^{NOA,U,*} - V_t^{NFO,G,*}$ neglects the debt tax shield;

- $V_t^{NOA,U,*} - V_t^{NFO,G,\tau}$ neglects the debt tax shield; the discount rates are not consistent;

- $V_t^{NOA,U,*} - V_t^{NFO,N,*}$ neglects corporate taxation in the discount rates, i.e. in the opportunity costs of investing and of borrowing; it can be consistent with $V_t^{NOA,L,*} - V_t^{NFO,G,*}$;

- $V_t^{NOA,U,*} - V_t^{NFO,N,\tau}$; the discount rates are not consistent;

- $V_t^{NOA,U,\tau} - V_t^{NFO,G,*}$ neglects the debt tax shield; the discount rates are not consistent;

- $V_t^{NOA,U,\tau} - V_t^{NFO,G,\tau}$ neglects the debt tax shield;

- $V_t^{NOA,U,\tau} - V_t^{NFO,N,*}$; the discount rates are not consistent;

- $V_t^{NOA,U,\tau} - V_t^{NFO,N,\tau}$ **seems the preferable solution**; this note highlights the merits of this solution;

- $V_t^{NOA,L,*} - V_t^{NFO,G,*}$ neglects corporate taxation in the discount rates, i.e. in the opportunity costs of investing and of borrowing, which generally implies a mis-valuation of levered equity; includes the textbook cases of APV and after-tax wacc; $V_t^{NFO,G,*} = NFO_t$ can be assumed;

- $V_t^{NOA,L,*} - V_t^{NFO,G,\tau}$; the discount rates are not consistent;

- $V_t^{NOA,L,*} - V_t^{NFO,N,*}$ double counts the debt tax shield;

- $V_t^{NOA,L,*} - V_t^{NFO,N,\tau}$ double counts the debt tax shield;

- $V_t^{NOA,L,\tau} - V_t^{NFO,G,*}$; the discount rates are not consistent;

- $V_t^{NOA,L,\tau} - V_t^{NFO,G,\tau}$ is consistent with the favourite solution $V_t^{NOA,U,\tau} - V_t^{NFO,N,\tau}$ when $V_t^{NOA,L,\tau} = V_t^{NOA,U,\tau} + V_t^{TS,\tau}$;

- $V_t^{NOA,L,\tau} - V_t^{NFO,N,*}$ double counts the debt tax shield;

- $V_t^{NOA,L,\tau} - V_t^{NFO,N,\tau}$ double counts the debt tax shield.

This enumeration of cases may perhaps be used by a judge in a court case litigation about the valuation of the debt tax shield.

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