Equity value and volatility

Moon Hoe Lee

Moonhlee@telus.net.

Edwards School of Business, University of Saskatchewan, Saskatoon, Saskatchewan, S7N 5C9, Canada.

Equity value and volatility

ABSTRACT

Common stock has equity value and volatility is present in stock prices. Equity value is created by the valuation of corporate and economic events. Volatility is a noisy component of the stock price. Both equity value and volatility are estimated using a derived first-order autoregressive process.

Demeaned stock prices are autoregressive. Common stock has equity value when demeaned stock prices are autoregressive.

The exponential rate and the volatility of the S&P 500 were very high at the start of the COVID-19 pandemic.

Keywords

First-order autoregressive process

Continuous time

Demeaned prices

Equity value

Exponential rate

Volatility

JEL Classification: G12

1 Introduction

I derive a first-order autoregressive process in continuous time for demeaned stock prices. The equity value of common stock and the volatility of stock prices are estimated by the above AR (1) process.

Demeaned stock prices are autoregressive. Common stock has equity value only when demeaned stock prices are autoregressive, i.e., when the exponential or discount rate is positive. An AR (1) equation is similar to a present-value equation. There is a present or discounted value of a cash flow when the discount rate is positive.

The equity value of a firm's common stock is derived from the valuation of corporate and economic events and is endogenous. Endogenous equity value is novel.

An example of price charts, which show the volatility of the S&P 500, is given in the well-known papers by Shiller (1981, 2014). The price chart shown in Shiller's (2014) article is an annual plot of the S&P 500 from 1871 to 2013. I test the AR (1) process on a more recent time series of the S&P 500.

Strong shocks to prices occur frequently. Strong shocks to prices can originate from trade tussles, imposition and lifting of tariffs, resolution of trade disputes, negotiations to free trade agreements, economic slowdown, economic growth, numerous corporate activities, domestic and foreign political unrests, threats of war, changes in Federal funds rate, changes in the prices of energy, changes in corporate income and personal income tax rates, budget standoffs, policy announcements, Federal Government shutdowns, U.S.-Mexico border crises, presidential elections, president's executive orders, congressional decisions, congressional elections, Supreme Court nominations and decisions, currency realignments, natural disasters, pandemics, and a host of other domestic and global shocks to the S&P 500.

External shocks have both positive and negative influences on stock prices and may explain the autoregression of demeaned stock prices.

Shiller (2014) discusses volatility and speculation on the S&P 500.

Current stock prices are speculative. I derive a formula for the relationship between variance and speculation. The current stock price is speculative because a speculation risk premium is positive.

An AR (1) process has a half-life. The half-life is the time a current demeaned stock price halves its value to the stationary mean stock price. An autoregressive random price movement occurs during the half-life.

In an AR (1) process, the implicit volatility $(1-\lambda)\nu$ is accounted for by an autoregressive random price movement due to random changes in the valuation of a firm's common stock. Speculation on the current stock price S_t produces residual volatility $\lambda\nu$. Speculation risk is the residual volatility of stock prices $\lambda\nu$. The speculation risk premium for a firm's common stock is $\gamma\,(\lambda\nu)^2$.

Black's (1990) measure of a risk premium is a function of variance. The speculation risk premium is also a function of variance. In the estimation of the speculation risk premium in this paper, the parameter γ is a risk parameter. A numerical value is not assigned to the risk parameter γ.

I estimate the parameters of demeaned stock prices as boundary-condition parameters and not as time-series parameters. Besides specifying the derived AR

(1) process differently from the standard AR (1) process, problems, which are associated with the stability of time-series parameters, are avoided.

2 AR (1) process with exp $(-\theta)$

Decompose a current stock price S_t into a mean stock price μ and a current random or demeaned stock price X_t as follows:

$$S_t - \mu = X_t \tag{1}$$

The differential of X_t follows a process given by:

$$dX_t = -\theta X_t\, dt + \boldsymbol{\sigma}\, dz \tag{2}$$

where $\theta > 0$ and $\boldsymbol{\sigma} > 0$.

$dX_t \sim N\{-\theta X_t\, dt, \boldsymbol{\sigma}^2\}$.

In the above process given by equation (2), X_t is continuously mean-reverting at a positive exponential rate θ. The mean of X_t is zero. The volatility is $\boldsymbol{\sigma}$ and dz~N (0,1).

A first-order autoregressive process is derived from the differential of X_t given by equation (2).

Change the variables to remove the drift.

$Z_t = \exp(\theta t)\, X_{t..}$

Then

$$dZ_t = \theta \exp(\theta t) X_t \, dt + \exp(\theta t) \, dX_t$$

$$dZ_t = \theta \exp(\theta t) X_t \, dt + \exp(\theta t) (-\theta X_t \, dt + \boldsymbol{\sigma} \, dz).$$

$$dZ_t = \exp(\theta t) \boldsymbol{\sigma} \, dz \quad (3)$$

The solution to the above stochastic differential equation is obtained by integrating both sides from s to t.

$$Z_t = Z_s + \boldsymbol{\sigma} \int_s^t \exp(\theta q) \, dz(q) \quad (4)$$

Reverse the change of variables.

$$X_t = \exp(-\theta t) Z_t$$

$$X_t = \exp(-\theta t) Z_s + \boldsymbol{\sigma} \int_s^t \exp(-\theta(t-q)) \, dz(q)$$

$$X_t = \exp(-\theta(t-s) X_s + \boldsymbol{\sigma} \int_s^t \exp(-\theta(t-q)) \, dz(q). \quad (5)$$

Set s = t−1 and let the volatility ν be proportional to the current stock price S_t and ν>0. The volatility ν is dimensionless and serves as a dimensionless measure of volatility.

$$X_t = \exp(-\theta) X_{t-1} + \nu S_t \int_{t-1}^t \exp(-\theta(t-q)) \, dz(q) \quad (6)$$

The point estimate of $\boldsymbol{\sigma}$ is $\upsilon\mu$, when υ is measured as the volatility proportional to the mean stock price. Empirically, the difference between υ

and ν is small.

The above first-order autoregressive process written in continuous time is evaluated to give

$$X_t = \phi X_{t-1} + \lambda \nu S_t dz \tag{7}$$

where $\phi = \exp(-\theta)$ and

$$\lambda = \int_{t-1}^{t} \exp\big(-\theta(t-q)\big)\, dz(q)$$

$$\lambda = \{\{1 - \exp(-2\theta)\}/2\theta\}^{½} \tag{8}$$

$$X_t \sim N\{\phi X_{t-1}, (\lambda\nu S_t)^2\}.$$

The parameter, λ, given by equation (8) is a number that falls within the range $0<\lambda<1$. An AR (1) process is a process with an autoregressive random price ($\lambda \neq 1$) and volatility ($\lambda \neq 0$).

Substitute X_t from equation (7) for X_t in equation (1) to give an AR (1) process for $(S_t - \mu)$.

The AR (1) process for $(S_t - \mu)$ is given by

$$S_t - \mu = \phi (S_{t-1} - \mu) + \lambda\nu S_t dz \tag{9}$$

$$(S_t - \mu) \sim N\{\phi (S_{t-1} - \mu), (\lambda\nu S_t)^2\}.$$

The parameter μ on both sides of the equation (9) is the mean of S_t from the same n observations. The sample size is n.

2.1 Equity value

$$S_t = \mu + \exp(-\theta)(S_{t-1} - \mu) + \lambda\nu S_t dz \quad (10)$$

A current stock price is the sum of the equity value and the volatility of stock prices. Equity value is derived from the valuation of corporate and economic events. A mean stock price is equity value. A stock transaction adds an increment or a decrement to the equity value. The exponential factor, $\exp(-\theta)$, discounts a lagged demeaned stock price to add equity value to a firm's common stock. Volatility is noise and does not add equity value to a firm's common stock.

External economic shocks keep the equity value current by changing the discounted lagged demeaned stock price. The valuation of a firm's common stock is updated with every stock transaction.

If it is assumed that at time $t-1$, X_{t-1} is a forecast of X_t, and the forecast is discounted by $\exp(-\theta)$, then the exponential factor, $\exp(-\theta)$, is a discount factor used in present-value calculations.

2.2 Estimation of parameters

The boundary conditions of the AR (1) process are $\theta>0$ and $\nu>0$. The two parameters θ, and ν are restricted from being negative by transforming each parameter to $\exp(-x)$.

The original parameters $\{\theta,\nu\}$ are exponentially transformed to new parameters $\{x_1,x_2\}$ by letting $\theta = \exp(-x_1)$ and $\nu = \exp(-x_2)$. Estimating a parameter θ by exponential transformation to parameter x_1 is novel. The original parameter $\mu = x_3$.

The three parameters $\{x_1, x_2, x_3\}$ are estimated jointly by maximizing a log-likelihood function (see Gauss). The log-likelihood function is based on the following log-density function:-

$$ld = \ln\{1/(2\pi)^{½} \lambda \nu S_t\} - \{S_t - \mu - \exp(-\theta)(S_{t-1} - \mu)\}^2 / 2(\lambda \nu S_t)^2 \quad (11)$$

The original parameters θ and ν can be recovered from their exponential functions. For example, the point estimate of θ is $\breve{\theta} \simeq \exp(-\breve{x})$. The subscript for x is suppressed. The maximum likelihood estimate of x is $\breve{x}$. The asymptotic standard error of $\exp(-\breve{x})$ is $|d\{\exp(-\breve{x})\}/d\breve{x}|$ x S.E. $(\breve{x})$, where S.E. $(\breve{x})$ is the standard error of $\breve{x}$. The t-statistic for $\breve{\theta}$ is 1/S.E. $(\breve{x})$. By eliminating the approximate mean $\exp(-\breve{x})$, the t-statistic for $\breve{\theta}$ depends on the standard error of the exponent $\breve{x}$ of the mean transformed parameter $\exp(-\breve{x})$ The value of the t-statistic is small if the value of the S.E. $(\breve{x})$ is high.

The exponent x of $\exp(-x)$ has a normal distribution with mean $\bar{x}$ and variance σ^2. In the derivation of a t-statistic for the point estimate $\breve{\theta}$, the numerical approximation of $\exp(-x)$ is used because it has a normal distribution with mean $\exp(-\bar{x})$ and variance $\exp(-2\bar{x})\,\sigma^2$. Note that $\exp(-\bar{x}) < \exp(-\bar{x} + ½\,\sigma^2)$. The test is to show the lower bound is greater than zero because a t-statistic can be computed for the lower bound. The exponential rate θ and the volatility ν are estimated in continuous time because the AR (1) process given by equation (6) is written in continuous time.

3 The half-life of an AR (1) process

3.1 Standard AR (1) process

Consider the following standard AR (1) process (Hamilton (1994))

$$y_t = c + \phi y_{t-1} + \varepsilon_t \quad (12)$$

The variable y_t is a random variable, c is a constant, $0 < \phi < 1$, $\varepsilon_t \sim N(0, \omega^2)$, and t = 1, 2, …T.

The stationary mean of an AR (1) process is $\bar{y} = c/(1-\phi)$.

Replace c by $\bar{y}(1-\phi)$ in equation (12) to give

$$y_t - \bar{y} = \phi(y_{t-1} - \bar{y}) + \varepsilon_t \quad (13)$$

Re-write the above equation equivalently to give

$$x_t = \phi x_{t-1} + \varepsilon_t \quad (14)$$

3.2 Half-life

The demeaned variable x_t is a measure of the value to the stationary mean $\bar{y}$. Assume at time t+h, the AR (1) process halves x_t to the stationary mean. Compute the half-life h such that

$$E_t(x_{t+h}) = \tfrac{1}{2} x_t.$$

From equation (14)

$E_t\ (x_{t+h}) = \phi^h\ x_t.$

So that

$\phi^h = ½.$

Taking natural logarithm

$$h = -\ln(2) / \ln(\phi) \qquad (15)$$

Setting $\phi = \exp(-\theta)$ and $\theta>0$, the half-life h varies inversely with θ the autoregressive speed. The half-life h is an alternative measure of the autoregressive speed. A longer half-life is equivalent to a slower autoregressive speed.

The demeaned variable x_t is autoregressive and volatile.

4 Data

The S&P 500 is obtained online from Yahoo! Finance. A sample of price data consists of the end of December closing S&P 500 of each year from 1990 to 2019. The annual prices are adjusted for stock splits. Yahoo! Finance performs the adjustment of stock splits.

A second sample of price data consists of the daily closing S&P 500. The prices are adjusted for stock splits. The daily price data are for two years, 2018 and 2019.

A quarterly sample of the daily closing S&P 500 is from March 2020 to May 2020. The sample period from March to May 2020 was in the early stage of the COVID-19 pandemic.

There were major shocks to market prices during the period. The major shocks to stock prices were the COVID-19 pandemic, the spread of the virus, the lockdown of the U.S. economy, and the passing of an initial U.S. congressional stimulus package of U.S. $2 trillion.

The ticker symbol for the S&P 500 is ^gspc. The website for Yahoo! Finance is given in the References.

5 Empirical evidence

According to the test equation (7), the current demeaned stock price is normally distributed. The test equation shows that the exponential rate θ is estimated from the exponent of the autoregressive coefficient exp $(-\theta)$. The estimated ν is the volatility of the current stock price.

The exponential rate θ estimated from 30 years of annual prices is an annual exponential rate. The volatility ν is annual volatility. The exponential rate θ estimated from one year of daily prices is a daily exponential rate in a given year. The estimated volatility ν is daily volatility.

The empirical research is to find out if the exponential rate θ and the volatility ν are statistically significant at the 5 % level of significance. The mean stock price μ is tested to see if it is a significant statistic.

Table 1 shows the point estimates of the annual parameters of an AR (1) process

Table 1 Estimates of an annual exponential rate and volatility

Security	Period	n	θ	ν	μ	h
S&P 500	1990-2019	30	0.4314	0.2009	1316	1.60
			(2.01)	(6.31)	(4.75)	

with autoregressive coefficient exp (–θ) and the half-life of an AR (1) process. The data consist of the end of December closing S&P 500 adjusted for stock splits from 1990 to 2019. The annual exponential rate θ and the volatility ν are restricted from being negative. The critical value of the t-statistic for θ and ν is 1.70 at the 5 % level of significance. The parameter μ is the mean S&P 500. The t-statistics for point estimates of the parameters are given within parentheses below the point estimates. The number of observations is n. The half-life of an AR (1) process is h years.

The annual exponential rate for the lagged demeaned S&P 500 as shown in Table 1 was 43.14 % from 1990 to 2019.

An application of the annual exponential rate is the estimation of an annual cost of equity. The annual cost of equity, c, which is a discount or capitalization rate, can be estimated from $\mu + \exp(-\theta)(S_{t-1} - \mu) = D/c$, where D is the current dividend per share. The equation is the equity/present-value model. The above equation represents the equity value (not price) and the present value of one share of the S&P 500 stock portfolio. For example, the average annual cost of equity for the S&P 500 stock portfolio was 2.78 % on December 31, 2019. The average annual dividend yield on the S&P 500 stock portfolio was 1.80 % on December 31, 2019. The yield on a 1-year U.S. Treasury bill was 1.51 % on December 31, 2019. If the dividend per share grows at 2 % a year (inflation rate), then the average annual cost of equity will be 4.78 %. Growth in dividend per share is a forecast, which is subject to uncertainty. Therefore, the cost of equity on December 31, 2019, was 2.78 %. The annual cost of equity for a firm, which does not pay a dividend, is zero. Gordon's (1959) dividend-yield models understate the cost of equity when the current stock price is greater than the equity value.

The annual volatility ν of the S&P 500 was 20.09 % from 1990 to 2019.

Table 2 shows the annual speculation risk premium for the S&P 500 stock portfolio from 1990 to 2019, and autoregressive random price movement measured as volatility $(1–\lambda)\nu$. The parameter ν is the annual volatility. The parameter λ is a function of θ the annual exponential rate. The annual volatility $\lambda\nu$ is the residual volatility of the AR (1) process. The annual speculation risk premium for the S&P 500 stock portfolio is $\gamma (\lambda\nu)^2$, and γ is a risk parameter. The level of annual speculation on the S&P 500 is inferred from the speculation risk premium. Observations are the end of December closing S&P 500 adjusted for stock splits. The number of observations is n.

Table 2 Annual speculation risk premium

Security	Period	n	$\lambda\nu$	$\gamma (\lambda\nu)^2$	$(1–\lambda)\nu$
S&P 500	1990-2019	30	0.1644	$\gamma\ 0.1644^2$	0.0365

From Table 2, the annual volatility of the S&P 500 of 3.65 % was due to random changes in the valuation of the S&P 500 stock portfolio. The annual volatility of the S&P 500 of 16.44 % was due to speculation on the S&P 500.

The stock price was more volatile (20.09 %) than the equity value (3.65 %). The extra volatility (16.44 %) was due to speculation.

The annual speculation risk premium for the S&P 500 stock portfolio was γ 0.1644^2.

The half-life of the current demeaned S&P 500 with annual prices from 1990 to

Table 3 Estimates of a daily exponential rate and volatility

Security	Period	n	θ	ν	μ	h
S&P 500	2018	251	0.0491 (2.42)	0.0125 (21.88)	2746 (68.09)	14
S&P 500	2019	252	0.0583 (2.63)	0.0159 (21.83)	2913 (52.54)	11
S&P 500	COVID-19 Pandemic	63	0.1248 (1.86)	0.0338 (10.55)	2773 (26.67)	5

2019 was 1.6 years.

Table 3 shows the point estimates of the daily parameters of an AR (1) process with autoregressive coefficient exp (–θ) and the half-life of an AR (1) process. The time series is one year, and three months for the COVID-19 pandemic sample. The daily exponential rate θ and the daily volatility ν are restricted from being negative. The critical value of the t-statistic for θ and ν is 1.64 at the 5 % level of significance for the samples of one year of the S&P 500 and 1.67 for the COVID-19 pandemic sample. The parameter μ is the mean S&P 500. The t- statistics for point estimates of the parameters are given within parentheses below the point estimates. Observations are daily closing S&P 500 adjusted for stock splits. The number of observations is n. The half-life of an AR (1) process is h days. Period: January-December 2018, January-December 2019, and March 2, 2020-May 29, 2020 for the COVID-19 pandemic sample.

An investment in the S&P 500 stock portfolio carries volatility risk and speculation risk. An increase in the exponential rate reduces the S&P 500 due to volatility risk.

During the COVID-19 pandemic, the exponential rate for the lagged demeaned S&P 500 rose to a very high rate (Table 3). The S&P 500 fell. The mean S&P 500 for the COVID-19 sample was 140 lower than the mean S&P 500 for the 2019 sample.

Table 4 shows the daily speculation risk premium for the S&P 500 stock portfolio over one year from 2018 to 2019, and autoregressive random price movement measured as volatility (1–λ)ν. The parameter ν is the daily volatility. The parameter λ is a function of θ the daily exponential rate. The COVID-19 pandemic sample; λ is a function of the daily exponential rate estimated over three months and ν is also estimated over three months. The daily volatility $\lambda\nu$ is the residual volatility of the AR (1) process. The speculation risk premium for the

Table 4 Daily speculation risk premium

Security	Period	n	$\lambda\nu$	$\gamma\,(\lambda\nu)^2$	$(1-\lambda)\nu$
S&P 500	2018	251	0.0122	$\gamma\ 0.0122^2$	0.0003
S&P 500	2019	252	0.0154	$\gamma\ 0.0154^2$	0.0005
S&P 500	COVID-19 Pandemic	63	0.0317	$\gamma\ 0.0317^2$	0.0021

S&P 500 stock portfolio is $\gamma\,(\lambda\nu)^2$, and γ is a risk parameter. The level of daily speculation on the S&P 500 is inferred from the speculation risk premium. Observations are daily closing S&P 500 adjusted for stock splits. The number of observations is n. Period: January-December 2018, January-December 2019, and March 2, 2020-May 29, 2020, for the COVID-19 pandemic sample.

During the COVID-19 pandemic, speculation on the S&P 500 rose and volatility was higher than usual (Table 4). Traders were pessimistic (dz=–1). The current S&P 500 was less than the equity value of the S&P 500 stock portfolio due to speculation risk. The current demeaned S&P 500 was negative in the early days of the COVID-19 pandemic.

From Table 3, the daily exponential rate θ for the lagged demeaned S&P 500 was 4.91 % in 2018 and 5.83 % in 2019. The daily exponential rates, not reported in Table 3, were 4.93 % in 2012, 3.72 % in 2013, 3.13 % in 2014, 6.66 % in 2015, 2.35 % in 2016 and 3.39 % in 2017, and were significant at the 5 % level.

The exponential rate θ is estimated as a boundary-condition rate and is stable. The standard error for θ is small in each of the ten years. The time-series parameter ϕ is not estimated. The volatility ν shown in Table 3 is estimated as a boundary-condition parameter and is stable. The standard error for ν is very small. The volatility ν is not estimated as a time-series parameter.

The exponential rate θ and the volatility ν are estimated as boundary-condition parameters over a thirty-year period and over a three-month COVID-19 pandemic. Standard errors for θ are small and standard errors for ν are very small.

A daily exponential factor, exp $(-\theta)$, serves the purpose of discounting a day's lagged demeaned S&P 500 and values the security. For example, the equity value of the S&P 500 stock portfolio on December 31, 2019, is given by μ + exp (–0.0583) $(S_{t-1} - \mu)$ = 2913 + 0.9433 (3221− 2913) = 3203. The current S&P 500 on December 31, 2019, was 3230.

The daily volatility ν of the S&P 500 shown in Table 3 was 1.25 % in 2018 and 1.59 % in 2019.

Table 3 shows the half-lives for the current demeaned S&P 500 over a sample period of one year. The half-lives for the current demeaned S&P 500, which ranged from 14 days in 2018 to 11 days in 2019, were relatively short. The daily volatility of the S&P 500 due to random changes in valuation was low, at about 0.05 %. The daily volatility of the S&P 500 due to speculation on the S&P 500 ranged from 1.22 % in 2018 to 1.54 % in 2019. The daily volatility of the S&P 500 was due mainly to speculation on the S&P 500.

As shown in Table 4, the speculation risk premiums for the S&P 500 stock portfolio from one year of data ranged from $\gamma\ 0.0122^2$ in 2018 to $\gamma\ 0.0154^2$ in 2019. The S&P 500 was speculative from 2018 to 2019.

Very major shocks occurred one after another during the sample period from March 2 to May 29, 2020. The World Health Organization declared the outbreak of COVID-19 a pandemic on March 11, 2020. The period, March-May 2020, is an exceptional period to study the market activity of the S&P 500.

The daily exponential rate θ for the lagged demeaned S&P 500 of 12.48 % at the start of the COVID-19 pandemic is estimated over three months (Table 3).

The daily volatility ν of the S&P 500 was 3.38 % at the start of the COVID-19 pandemic. Both the exponential rate and the daily volatility were very high at the start of the COVID-19 pandemic.

The daily volatility due to speculation on the S&P 500 was 3.17 % during the early stage of the COVID-19 pandemic. The daily speculation risk premium for the S&P 500 stock portfolio in the COVID-19 pandemic market was $\gamma\ 0.0317^2$. The S&P 500 was over speculative in trading at the start of the COVID-19 pandemic.

The half-life of 5 days was very short. The daily volatility of the S&P 500 due to random changes in the valuation of the S&P 500 stock portfolio was 0.21 %. The daily volatility of the S&P 500 due to speculation on the S&P 500 was 3.17 %.

The daily volatility of the S&P 500 was due mainly to speculation on the S&P 500.

As shown in Tables 1 and 3, the mean S&P 500 is a significant statistic of the current demeaned S&P 500. The mean S&P 500 is equity value. The discounted lagged demeaned S&P 500 is added equity value. Common stock has equity value is the principal finding.

6 Conclusion

I derive a first-order autoregressive process for the current demeaned stock price. The current demeaned stock price shows the composition of a current stock price. A current stock price is the sum of the equity value and the volatility of stock prices. A current stock price has an added value, which forms part of the equity value. The added value to a firm's common stock is computed by discounting a lagged demeaned stock price.

Common stock has equity value when demeaned stock prices are autoregressive, i.e., when the exponential or discount rate is positive. Equity value is created by the valuation of corporate and economic events. A first-order autoregressive process is the test equation.

The above AR (1) process was tested on the S&P 500. The empirical results show that the mean S&P 500 and the discounted lagged demeaned S&P 500 are equity values. The exponential or discount rate is endogenous.

Some applications of determining the equity value of common stock are estimating an annual cost of equity, estimating the gains in synergies of mergers, any gain in the equity value of stock splits, and the gains in the equity value of the breakup of conglomerates.

References

Black F. (1990). Mean reversion and consumption smoothing. Review of Financial Studies, 3, 107-114.

GAUSS Maximum likelihood. Maple Valley, Washington, Aptech Systems Inc.

Gordon M.J. (1959). Dividends, earnings, and stock prices. Review of Economics and Statistics, 41, 99-105.

Hamilton J.D. (1994). Time series analysis. Princeton, Princeton University Press.

Shiller R.J. (1981). Do stock prices move too much to be justified by subsequent changes in dividends? American Economic Review, 71, 421-436.

Shiller R.J. (2014). Speculative asset prices. American Economic Review, 104, 1486-1517.

https://finance.yahoo.com/quote/%5EGSPC/history/

Yahoo! Finance. *http://finance.yahoo/*