A NOTE ON GENERALISED RESIDUAL INCOME VALUATIONS

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Abstract

Ohlson and Johannesson (2016) and Ohlson (2022) present generalisations of abnormal earnings growth (AEG) valuations (GAEGV) and Lai (2020) applies GAEGV to explain valuation multiples other than price to earnings. This paper presents generalisations of residual income valuation (GRIV) to further explain valuation multiples. Unlike GAEGV, GRIV encompass hybrid valuation models. Both GRIV and GAEGV anchor the valuation on a multiple, but while GAEGV rely on forecasts of a generalised version of AEG, GRIV rely on forecasts of a generalised version of residual income. GRIV can reconcile analysts' use of the valuation multiples of comparable companies, with fundamental value estimates based on forecasts of accounting variables seemingly unrelated to the said multiples of comparable companies.

Some GRIV naturally explain cash flow based valuation multiples by discounting either abnormal dividends growth or abnormal operating cash flow growth.

Key words: valuation formulae, valuation multiples, RIV, AEG.

JEL classification: G12; G13.

1 Introduction

Ohlson and Johannesson (2016), in short OJ (2016), and Ohlson (2022) generalise the abnormal earnings growth (AEG) valuation of Ohlson (2005) and Ohlson and Juettner-Nauroth (2005), so that the price to forward earnings multiple equals the long term mean of the same multiple plus a transitory mean reverting component. Lai (2020) adapts the valuation of OJ (2016) to explain other valuation multiples, such as enterprise value (EV) to EBITDA and EV to sales. We hereafter collectively refer to these insightful models as generalised AEG valuations (GAEGV).

This paper presents new valuations that further explain a variety of valuation multiples, including cash flow based multiples, even through forecasts of accounting variables that do not define the valuation multiple to be explained. We refer to these valuations collectively as generalised residual income valuations (GRIV). Many of these valuations are "hybrid" in a spirit similar to the empirically successful hybrid valuation of Gao and others (2019). Gao and others (2019) present a hybrid valuation that forecasts and discounts dividends over a finite forecast horizon, and then uses the GAEGV of OJ (2016) to estimate the terminal value of equity at the end of the forecast horizon. The GRIV in this paper encompass a variety of other hybrid valuation models.

Residual income valuation (RIV) is typically used to explain the price to book multiple, but GRIV can also explain many other valuation multiples, including price to dividends, EV to net operating assets, EV to operating income, EV to EBITDA, EV to free cash flow, EV to operating cash flow, EV to sales. GRIV are all equivalent to discounting dividends in equity valuations or to discounting free cash flows in enterprise valuations. While the GAEGV of Lai (2020) rely on forecasts of generalisations of AEG, GRIV rely on forecasts of generalisations of residual income.

Another difference between GRIV and GAEGV is that GRIV encompass hybrid valuations. While both GRIV and GAEGV can explain a variety of valuation multiples, GAEGV explain a multiple through forecasts based on the variable that defines the multiple. For example in Lai (2020) the EV to EBITDA multiple is explained through forecasts of abnormal growth of EBITDA, or the EV to sales multiple is explained through forecasts of abnormal growth of sales. Instead hybrid GRIV can explain a multiple through forecasts of an accounting variable that does not define the multiple. For example GRIV can explain the EV to sales multiple through forecasts of operating income. Similarly GRIV can explain the EV to operating cash flow multiple through forecasts of operating income. In other words GRIV can reconcile most of the popular valuation multiples with most of the popular valuation models and with hybrid valuation models too. This means reconciling valuations based on the multiples of comparable companies with valuations based on fundamental analysis, even in cases whereby the explicit link between the multiple and the fundamental valuation has so far been overlooked.

GRIV can also naturally explain valuation multiples based on cash flows. They can explain price to dividends through forecasts of abnormal dividends growth. They can explain the popular EV to operating cash flow multiple through forecasts of abnormal operating cash flow growth.

The paper is organised as follows. The next section reviews the GAEGV of OJ (2016), of Ohlson (2022) and of Lai (2020). Then GRIV and their many variants are introduced.

2 Generalised abnormal earnings growth valuations (GAEGV)

OJ (2016), Lai (2020) and Ohlson (2022) present what we refer to as GAEGV. GAEGV can parsimoniously explain various valuation multiples. To summarise GAEGV the notation is:

- $V_t \in \left\{V_t^e, V_t^f\right\}$; V_t^e is equity value at time t; V_t^f is enterprise value (EV) at time t; this notation means that V_t may be equal to V_t^e or to V_t^f;

- m is the long term mean level to which the valuation multiple V_t/x_{t+1} tends to revert;

- x_{t+1} is an accounting variable on which the valuation anchors;

- $x_{t+1} \in \{x^e_{t+1}, OI_{t+1}, S_{t+1}, EBITDA_{t+1}\}$; x^e_{t+1} denotes earnings, OI_{t+1} denotes operating income, S_{t+1} denotes sales, $EBITDA_{t+1}$ denotes EBITDA during $[t, t+1]$;

- $r \in \{r_e, r_f\}$; r_e is equity cost of capital; r_f is the cost of capital of the enterprise;

- $z_{t+1} \in \{d_{t+1}, C_{t+1} - I_{t+1}\}$; d_{t+1} denotes net dividends, C_{t+1} denotes operating cash flow and I_{t+1} denotes capital expenditure during $[t, t+1]$.

We assume $t = 0$ is the valuation date. This aids the later comparison with GRIV models, which are convenient to present under the same assumption. Then we can summarise GAEGV as:

a) $V_t = \frac{z_{t+1}+V_{t+1}}{1+r}$ for $t \geq 0$; this means GEAGV are equivalent to discounting cash flows, be they dividends or free cash flows;

b) $V_0 = m \cdot x_1 + \frac{m \cdot v_2}{1+r-h}$ and $v_2 = x_2 + z_1/m - (1+r) \cdot x_1$; $m > 0$ is a constant such that $\frac{1}{m} < r$;

c) $v_{t+1} = h \cdot v_t$ for $t \geq 2$; v is GAEG (generalised AEG) and an information variable; h is a constant such that $1 + r > h$ and $0 < h < 1$.

Any two of the above three statements a), b), c) imply the remaining statement. x_1 and v_2 are forecasts. GAEGV assume that GAEG is expected to revert to zero. When $m = \frac{1}{r}$ and $x_1 = x^e_1$ GAEGV become the well known AEG valuation of Ohlson (2005) and Ohlson and Juettner-Nauroth (2005).

GAEGV have not been but could be used to explain also cash flow multiples, such as price to dividends or EV to operating cash flow. This can be done by setting

$$V_0 = m \cdot z_1 + m \cdot \frac{z_2 + z_1/m - (r+1) \cdot z_1}{1 + r - h} \tag{1}$$

but in this case GAEG becomes $z_2 + z_1/m - (r+1) \cdot z_1$ and is not easy to interpret unless $1/r = m$.

The GAEGV for equity valuation that have been proposed assume $V_t = V_t^e$, $z_t = d_t$, $r = r_e$ and $x_t = x_t^e$ as in OJ (2016) and Ohlson (2022). Then m is the long term mean of the price to forward earnings ratio. We refer to v_2 as generalised AEG (GAEG), since when $\frac{1}{m} = r$ GAEG coincides with AEG and V_t equals AEG valuation. Thus OJ (2016) can be viewed as a generalisation of Ohlson (2005) and Ohlson and Juettner-Nauroth (2005) to the case whereby $m > \frac{1}{r_e}$.

Following Lai (2020), the GAEGV for enterprise valuation (EV) assume $V_t = V_t^f$, $z_t = C_t - I_t$, $r = r_f$ and $x \in \{OI, S, EBITDA\}$. m is the long term mean of V_t^f / x_{t+1}. For example GAEGV that anchor on operating income assume $x = OI$ such that

$$V_0^f = m \cdot OI_1 + \frac{OI_2 + \frac{1}{m}(C_1 - I_1) - (1 + r_f) \cdot OI_1}{1 + r_f - h}$$

where m is the long term mean of V_t^f / OI_{t+1}. OI_{t+1} can be negative, in which case V_t^f / OI_{t+1} is difficult to interpret. GAEGV that anchor on sales assume $x = S$ and have the advantage that S is less likely to be negative than OI. GAEGV that anchor on EBITDA assume $x = EBITDA$. Again $EBITDA_{t+1}$ is often less likely to be negative than is OI_{t+1}.

A merit of GAEGV is that they provide easy estimates of the implied cost of capital of equity or of the enterprise.

Remark 1 *If* V_0*,* m*,* $(1 + r_f - h) > 0$*,* $m \cdot (x_2 - x_1 \cdot h) + z_1 \neq 0$*, and if we set* $V_0 = P_0$ *where* P_0 *is the time* 0 *market price of equity or of the enterprise, as the case may be, the implied cost of capital is*

$$r = \frac{m \cdot (x_2 - x_1 \cdot h) + z_1}{V_t} + h - 1.$$

Of course GAEGV can also be used to reverse engineer h rather than r.

3 Generalised residual income valuations (GRIV)

This section introduces GRIV, which seem to provide some new flexibility to explain valuation multiples. We still assume that the valuation time is 0. The common feature of GRIV is the following algebric tautology for all times $t \geq 0$

$$V_t = \frac{z_{t+1} + V_{t+1}}{1 + r} = \Theta_t + \frac{\theta_{t+1} - \Theta_t \cdot r + V_{t+1} - \Theta_{t+1}}{1 + r} \tag{2}$$

$$\Theta_{t+1} = \Theta_t + \theta_{t+1} - z_{t+1}$$

where

$$V_t \in \left\{V_t^e, V_t^f\right\}$$

$$z_{t+1} \in \{d_{t+1}, C_{t+1} - I_{t+1}\}$$

$$r \in \{r_e, r_f\}.$$

Tautology 2 entails that all GRIV are equivalent to discounting the sequence of cash flows $z_{t+1}, z_{t+2}, z_{t+3}, ..$. Residual income valuation (RIV) is a special case of tautology 2 when $V_t = V_t^e$, $r = r_e$, $\Theta_t = B_t$ where is the book value of equity at t, $\theta_{t+1} = x_{t+1}^e$ and $z_{t+1} = d_{t+1}$. In this case Θ_{t+1} and Θ_t are linked by the clean surplus relation. However RIV is only a special case of tautology 2. For example in GRIV Θ_t may be a multiple of θ_{t+1} or of z_{t+1}. Moreover in GRIV each term of the sequence $\theta_{t+1}, \theta_{t+2}, \theta_{t+3}, ..$ can be a different type of accounting variable. The GRIV we consider assume

$$\Theta_0 = y_{i,1} \cdot m$$

$$\theta_1 = y_{j,1}$$

$$\theta_t = y_{k,t} \text{ for } t \geq 2$$

and explain the difference between the time 0 fundamental value V_0 and the valuation anchor $y_{i,1} \cdot m$. m is again a constant and $y_{i,1}$ is an accounting number expected to be known at time 1 or already known at time 0. $y_{i,1} \cdot m$ is itself an estimate of fundamental value derived from comparable companies, as explained below. Since $y_{i,1}$, $y_{j,1}$ and $y_{k,2}$ may all be different accounting variables, GRIV encompass hybrid valuation models, unlike GAEGV.

GRIV explain $V_0 - m \cdot y_{i,1}$ and the GAEGV of Lai (2020) and of OJ (2016) explain $V_0 - m \cdot x_1$. In both cases the forecast horizon tends to be shorter and the forecast effort tends to be less than that of a valuation that estimates V_0. $m \cdot y_{i,1}$ or $m \cdot x_1$ are components of fundamental value, often substantials ones, that require no forecast beyond beyond time 1, while m is estimated from observations of comparable companies at time 0. This explains much of the appeal of GRIV and GAEGV.

Remark 2 *The GRIV we consider have an explicit foreacast horizon of two periods so that*

$$V_0 = m \cdot y_{i,1} + \frac{RI_{j,1}}{1+r} + \frac{RI_{k,2}}{(1+r)(1+r-h)} \tag{3}$$

$$RI_{j,1} = y_{j,1} - y_{i,1} \cdot m \cdot r$$

$$RI_{k,2} = y_{k,2} - (y_{i,1} \cdot m + y_{j,1} - z_1) \cdot r$$

$$RI_{k,t+1} = h \cdot RI_{k,t}, t \geq 2,\ 0 < h < 1 + r.$$

These GRIV rely on the forecasts $y_{j,1}$, $y_{k,2}$ and $y_{i,1}$. This paper confines its attention to cases whereby either $y_j = y_i$ or $y_j = y_k$, which appear of greater interest, thus overlooking cases whereby $y_i \neq y_j \neq y_k$. We refer to RI as generalised residual income, which only coincides with residual income when $m = 1$, $y_{i,1} = B_0$, $y_{j,1} = x_1^e$ and $y_{k,t} = x_t^e$ for $t \geq 2$. It often helps to interpret $m \cdot y_{i,1}$ as "revalued" equity book value at time 0. Note that for $t \geq 3$

$$RI_{k,t} = y_{k,t} - \left[y_{i,1} \cdot m + y_{j,1} - z_1 + \sum_{l=2}^{t-1} (y_{k,l} - z_l)\right] \cdot r.$$

Again it often helps to interpret the term in square brackets as equity book value at time $t-1$ after the revaluation of equity book value at time 0.

If $0 < h < 1$, then

$$\lim_{t \to \infty} RI_{k,t} \to 0$$

$$\lim_{t \to \infty} \left(V_t - \left[y_{i,1} \cdot m + y_{j,1} - z_1 + \sum_{l=2}^{t} (y_{k,l} - z_l)\right]\right) \to 0.$$

However we do not impose the condition $h < 1$ in the GRIV of equations 3. This constraint may be appropriate for some GRIV, but not for others. The reason is that, unlike in GAEGV, in GRIV m cannot always be interpreted as the long term level of $V_t/y_{i,t+1}$, i.e. as $\lim_{t\to\infty} V_t/y_{i,t+1}$. Instead we interpret m as an estimate of the multiple $V_0/y_{i,1}$ computed as an average or median of the same multiple of comparable companies. In this sense m is the "normal" level of the said valuation multiple, much as in Lai (2020).

m may also equal $P_0/y_{i,1}$, where P_0 is the market price of equity or of enterprise at time 0. However in this paper we do not pursue this interpretation of m. This interpretation implies that P_0 may differ from fundamental value V_0.

We can re-write equation 3 as

$$V_0 = \frac{y_{j,1}}{r} + \frac{y_{k,2} - y_{j,1} - (y_{j,1} - z_1) \cdot r}{r\,(1 + r - h)}. \tag{4}$$

Equation 4 is akin to AEG valuation, and, unlike GRIV, does not explictly explain $V_0 - y_{i,1} \cdot m$. Equation 3 implies equation 4, but equation 4 does not imply equation 3. To make economic sense, the AEG valuation literature implicitly imposes the restriction $y_k = y_j = x^e$ in equation 4. GRIV do not need this restriction. GRIV encompass AEG valuation. However, insofar as $m \neq 1/r$ GAEGV differ from AEG valuation and are not special cases of GRIV.

As GAEGV, also GRIV in equation 3 can provide tractable implied cost of capital estimates.

Remark 3 *If we set* $P_0 = V_0$ *where* V_0 *is as per equation 3, if* $V_0 > 0$ *and if*

$b^2 \geq 4cV_0$, *then*

$$r = \frac{1}{2V_0}\left(-b \pm \sqrt{b^2 - 4cV_0}\right)$$

$$b = V_0 \cdot (2-h) - z_1$$

$$c = (V_t - y_{i,1} \cdot m - y_{j,1}) \cdot (1-h) - y_{k,2}.$$

r solves the equation $V_0 r^2 + br + c = 0$.

3.1 GRIV for equity valuation

In the GRIV of equation 3 that value equity:

- $V = V^e$ and $r = r_e$ and $z = d$;

- $y_{i,1} \in \{x_1^e, d_1, B_0\}$; B_0 denotes the book value of equity at time 0; the cases $y_{i,1} \in \{x_0^e, d_0, B_1\}$ are beyond the scope of this paper;

- $y_{j,1} \in \{x_1^e, d_1\}$;

- $y_{k,t} \in \{x_t^e, d_t\}$ for $t \geq 2$.

When $z = d$, $y_{i,1} = 0$, $y_{j,1} = x_1^e$, $y_{k,t} = x_t^e$ for $t \geq 2$ we recover the following special case of abnormal earnings (AE) valuation by Realdon (2019)

$$V_0 = \frac{RI_1}{1+r_e} + \frac{1}{1+r_e} \cdot \frac{RI_2}{1+r_e-h}$$

$$RI_1 = x_1^e$$

$$RI_t = x_t^e - \sum_{l=1}^{t-1}(x_l^e - z_l) \cdot r_e,\ t \geq 2.$$

3.2 GRIV for Enterprise Valuation (EV)

In the GRIV of equation 3 that compute EV:

- $V = V^f$ and $r = r_f$ and $z = C - I$;

- $y_{i,1} \in \{OI_1, C_1, C_1 - I_1, EBITDA_1, S_1, NOA_0\}$; NOA_0 denotes the book value of net operating assets at time 0; the cases $y_{i,1} \in \{OI_0, C_0, C_0 - I_0, EBITDA_0, S_0, NOA_1\}$ are beyond the scope of this paper;

- $y_{j,1} \in \{OI_1, C_1, C_1 - I_1, EBITDA_1, S_1\}$;

- $y_{k,t} \in \{OI_t, C_t, C_t - I_t, EBITDA_t, S_t\}$ for $t \geq 2$.

When $m = 1$, $y_{i,1} = NOA_0$, $y_{j,1} = OI_1$ and $y_{k,t} = OI_t$ for $t \geq 2$ we obtain "classic" residual operating income valuation (ROIV) under the assumption of constant growth in residual operating income.

When $z = C - I$, $y_{i,1} = 0$, $y_{j,1} = OI_1$ and $y_{k,t} = OI_t$ for $t \geq 2$ we obtain the EV variant of AE valuation by Realdon (2019) under the assumption of constant growth in abnormal operating income.

3.3 GRIV whereby $y_{j,1} = z_1$ and $y_{k,2} = z_2$

We now focus on the GRIV in equation 3 that assume that:

- $y_{j,1} = z_1$ and $y_{k,2} = z_2$;

- $y_{i,1}$ may equal z_1 or z_0 or 0 or another accounting variable.

These GRIV assume that z is free cash flow when $V = V^f$ or that z is net dividends when $V = V^e$, so that

$$V_0 = m \cdot y_{i,1} + \frac{RI_{j,1}}{1+r} + \frac{RI_{k,2}}{(1+r)(1+r-h)}$$

$$RI_{j,1} = z_1 - y_{i,1} \cdot m \cdot r$$

$$RI_{k,2} = z_2 - y_{i,1} \cdot m \cdot r.$$

These GRIV only forecast cash flows, except for the forecast of $y_{i,1}$, and naturally explain cash flow multiples, but not only cash flow multiples.

For example, when $y_{i,1} = z_1$ and $z = d$ these GRIV can explain the equity price to forward dividends multiple through an estimate m of the normal level of the said multiple from comparable companies. This estimate would provide the valuation anchor $d_1 \cdot m$, while $V_0^e - m \cdot d_1$ is the deviation of the fundamental value of the company in question from the said valuation anchor. Such deviation is explained by forecasts of future "residual dividends" as opposed to plain dividends. The appeal of this valuation, when compared with standard discounted dividends valuation, is that it relies less on future forecasts and terminal value estimates, because it anchors on the current normal equity price to forward dividends multiple inferred from comparable companies.

Similarly, when $y_{i,1} = C_1 - I_1$ and $z = C - I$ the GRIV in this section can explain the EV to forward free cash flow multiple through an estimate m of the normal level of the said multiple from comparable companies, which would provide the valuation anchor $(C_1 - I_1) \cdot m$. Then $V_0^f - m \cdot (C_1 - I_1)$ would be explained by forecasts of residual free cash flows of the company in question. Again the appeal of this valuation, when compared with standard discounted free cash flow valuation, is that it relies less on future forecasts and terminal value estimates, because it anchors on the current normal EV to forward free cash flow multiple inferred from comparable companies.

It seems useful to compare the GRIV in this section with GAEGV that explain cash flow multiples. In equation 1 we noted that also GAEGV could be used to explain cash flow multiples. In equation 1 $v_2 = z_2 + z_1/m - (r+1) \cdot z_1$, but then v_2 is not easy to interpret unless $1/r = m$. This interpretation issue does not affect GRIV. Moreover equation 1 only explains $V_0 - m \cdot z_1$, while the GRIV in this section explain $V_0 - m \cdot y_{i,1}$ and $y_{i,1}$ may differ from z_1. In this sense the GRIV of this section seem more general than the GAEGV of equation 1.

For example $y_{i,1}$ could be the equity a book value at time 0, i.e. B_0. Then $B_0 \cdot m$ would be the valuation anchor computed from the price to book multiple of comparable companies. Then $V_0^e - B_0 \cdot m$ can be explained by forecasts of "residual dividends" of the company in question, while $B_0 \cdot m$ requires no forecast. The forecasting effort and horizon to estimate $V_0^e - B_0 \cdot m$ tends to be less than that to estimate V_0^e. Herein lies the appeal of this valuation when compared with discounted dividends valuation whereby, other things equal, $y_{i,1} = 0$.

Similarly $y_{i,1}$ could be the book value of net operating assets at time 0, i.e. NOA_0, and m an estimate of the normal level of the EV to NOA_0 multiple from comparable companies. Then $V_0^f - NOA_0 \cdot m$ can be explained by forecasts of "residual free cash flows" of the company in question, while $NOA_0 \cdot m$ requires no forecast. Again the forecasting effort and horizon to estimate $V_0^f - NOA_0 \cdot m$ tends to be less than that to estimate V_0^f. Herein lies the appeal of this valuation when compared with standard discounted free cash flow valuation whereby, other things equal, $y_{i,1} = 0$.

We next focus on various other special cases of GRIV as per equation 3. The GRIV we consider assume $y_i = y_j = y_k$ or $y_i \neq y_j = y_k$ or $y_i = y_j \neq y_k$. We do no consider GRIV whereby $y_i \neq y_j \neq y_k$.

3.4 GRIV whereby $y_i = y_j = y_k$

We now review GRIV as per equation 3 whereby $y_i = y_j = y_k$ and whereby:

- for equity valuation $V = V^e$, $r = r_e$, $z = d$, $y_i \in \{x^e, d\}$;

- for enterprise valuation $V = V^f$, $r = r_f$, $z = C-I$, $y_i \in \{S, C, EBITDA, (C-I), OI\}$.

The case where $y_i = y_j = y_k = d$ was reviewed in the previous section. Instead when $y_i = y_j = y_k = x^e$ equity valuation as per equation 3 becomes

$$V_0^e = x_1^e \cdot \frac{m+1}{1+r_e} + \frac{x_2^e - (x_1^e \cdot m + x_1^e - d_{t+1}) \cdot r_e}{(1+r_e)(1+r_e-h)}.$$

If $m = 1/r_e$, this equity valuation becomes $V_0^e = \frac{x_1^e}{r_e} + \frac{AEG_2}{(1+r_e)(1+r_e-h)}$, since $AEG_{t+1} = h \cdot AEG_t$ for $t \geq 3$. Note that this equity valuation differs from the GAEGV of OJ (2016).

When $y_i = y_j = y_k$ enterprise valuation as per equation 3 can be summarised as

$$V_0^f = y_{i,1} \cdot m + \frac{y_{i,1} - y_{i,1} \cdot m \cdot r_f}{1+r_f} + \frac{y_{i,2} - (y_{i,1} \cdot m + y_{i,1} - (C_1 - I_1)) \cdot r_f}{(1+r_f)(1+r_f-h)}$$
$$y_i \in \{S, C, EBITDA, (C-I), OI\}.$$

This valuation entails that if $y_{i,2} > (y_{i,1} \cdot m + y_{i,1} - (C_1 - I_1)) \cdot r_f$ then the multiple $V_0^f / y_{i,1}$ will be higher than $\frac{m+1}{1+r_f}$. In this valuation m can be estimated from the $V_0^f / y_{i,1}$ multiple of comparable companies.

When $y_i = y_j = y_k = OI$, m can be estimated from the V_0^f / OI_1 multiple of comparable companies. Then $V_0^f - OI_1 \cdot m$ is explained by forecasts of some kind of residual operating income of the company in question.

When $y_i = y_j = y_k = C - I$ equation 3 becomes a seemingly new type of discounted free cash flow (DFC) valuation, i.e.

$$V_0^f = (C_1 - I_1) \cdot \frac{m+1}{1+r_f} + \frac{(C_2 - I_2) - (C_1 - I_1) \cdot m \cdot r_f}{(1+r_f)(1+r_f-h)}.$$

This valuation shows that if $C_2 - I_2 > (C_1 - I_1) \cdot m \cdot r_f$ then the EV to forward free cash flow multiple $V_t^f / (C_1 - I_1)$ will be higher than $\frac{m+1}{1+r_f}$. In this valuation m can be estimated from the $V_0^f / (C_1 - I_1)$ multiple of comparable companies.

When $y_i = y_j = y_k = C$ equation 3 becomes a seemingly new type of discounted cash flow valuation, i.e.

$$V_0^f = C_1 \cdot \frac{m+1}{1+r_f} + \frac{C_2 - (C_1 \cdot m + I_1) \cdot r_f}{(1+r_f)(1+r_f-h)}.$$

Again this valuation shows that if $C_2 > (C_1 \cdot m + I_1) \cdot r_f$ the EV to forward operating cash flow multiple V_0^f / C_1 will be higher than $\frac{m+1}{1+r_f}$. In this valuation m can be estimated from the V_0^f / C_1 multiple of comparable companies. $C_2 - (C_1 \cdot m + I_1) \cdot r_f$ is a kind of residual operating cash flow during $[t+1, t+2]$. This is an example of Abnormal Operating Cash Flow Valuation (ACV), which is presented more extensively below.

3.5 GRIV whereby $y_j = y_i$ and $y_k \neq y_j$

We now focus on GRIV as per equation 3 whereby $y_j = y_i$ and $y_k \neq y_j$ and whereby:

- for equity valuation $V = V^e$, $r = r_e$, $z = d$, $y_i, y_k \in \{x^e, d\}$;
- for enterprise valuation $V = V^f$, $r = r_f$, $z = C - I$, $y_i, y_k \in \{S, C, EBITDA, (C - I), OI\}$.

These GRIV encompass those whereby $y_i = y_j = y_k$, but the specific theme of these valuations is that $y_{i,1}$ can differ from $y_{k,2}$. In these valuations

$$V_0 = y_{i,1} \cdot \frac{m+1}{1+r} + \frac{RI_{k,2}}{(1+r)(1+r-h)}$$

$$RI_{j,1} = y_{i,1} - y_{i,1} \cdot m \cdot r$$

$$RI_{k,2} = y_{k,2} - (y_{i,1} \cdot m + y_{i,1} - z_1) \cdot r.$$

These valuations can also be written as

$$V_0 = y_{i,1} \cdot \frac{(1+m)(1-h)}{(1+r)(1+r-h)} + \frac{y_{k,2} + z_1 \cdot r}{(1+r)(1+r-h)}.$$

The first term in this formula anchors the valuation on $y_{i,1}$ times a constant, while the second term estimates a "terminal value" proportional to the forecast $y_{k,2}$.

For example, an enterprise valuation can anchor on a multiple of forward sales computed from comparable companies by setting $y_{i,1} = S_1$. This popular multiple has the merits that sales is almost always a non-negative number, which is condition for such multiple to be meaningful, and that it is insensitive to accounting policies that measure expenses in the income statement. At the same time the valuation can forecast operating income of the company in question by setting $y_{k,2} = OI_2$, as the latter is a better measure of value added than sales, and therefore arguably more relevant than sales for valuation purposes. This type of EV reconciles the fact that analysts do refer to the EV to sales multiple, and that they do forecast operating income in fundamental valuations. Setting $y_{i,1} = OI_1$ would have the drawback that operating income may be negative, in which case the EV to operating income multiple would be difficult to interpret.

Most of these valuations need the three forecasts $y_{i,1}, z_1, y_{k,2}$. Instead when we anchor the valuation on a multiple of free cash flows $y_{i,1} = z_1$ and we only need the two forecasts $z_1, y_{k,2}$ and

$$V_0 = z_1 \cdot \frac{(1+m)(1-h)+r}{(1+r)(1+r-h)} + \frac{y_{k,2}}{(1+r)(1+r-h)}.$$

One such example is an enterprise valuation that anchors on a multiple of forward free cash flows so that $z_1 = C_1 - I_1$. At the same time the valuation can forecast operating income of the company in question by setting $y_{k,2} = OI_2$, as the latter is often regarded as a more persistent measure of value added than free cash flows. This type of EV reconciles analysts' use of the EV to free cash flow multiple with their forecast operating income in a valuation.

3.6 GRIV whereby $y_j \neq y_i$ and $y_k = y_j$

Now we review GRIV as per equation 3 whereby $y_j \neq y_i$ and $y_k = y_j$ and whereby:

- for equity valuation $V = V^e$, $r = r_e$, $z = d$, $y_k \in \{x^e, d\}$;

- for enterprise valuation $V = V^f$, $r = r_f$, $z = C - I$, $y_i, y_k \in \{S, C, EBITDA, (C-I), OI\}$.

The models that follow are special cases of these GRIV. Also these GRIV encompass cases in which $y_i = y_j = y_k$.

3.6.1 When the anchor is proportional to equity book value

When $y_{i,1} = B_0 \cdot m$ and $y_k = y_j = x^e$, the GRIV of equation 3 become

$$V_0^e = B_0 \cdot m + \frac{x_1^e - B_0 \cdot m \cdot r_e}{1+r_e} + \frac{x_2^e - (B_0 \cdot m + x_1^e - d_1) \cdot r_e}{(1+r_e)(1+r_e-h)}.$$

$B_0 \cdot m$ can be interpreted a revalued book value of equity at time 0, and $B_0 \cdot m + x_1^e - d_1$ as book value of equity at time 1 after the said revaluation. x_1^e denotes actual reported earnings, unaffected by the revaluation to $B_0 \cdot m$ of the book value of equity at time 0. This valuation is equivalent to discounting dividends even if earnings are not adjusted to account for the effects of the said revaluation at time 0. For example depreciation and amortisation in the income statement need not be adjusted.

Then $(x_1^e - B_0 \cdot m \cdot r_e)$ and $(x_2^e - (B_0 \cdot m + x_1^e - d_1) \cdot r_e)$ can respectively be interpreted as residual income at times 1 and 2 after the said revaluation at time 0. If such residual income is positive, then $V_0^e / B_0 > m$. This valuation can explain the price to book ratio V_0^e / B_0. m can be estimated from the price to book multiple of comparable companies, and forecasts of (residual) earnings can explain $V_0^e - B_0 \cdot m$, which the difference between the fundamental value of equity and the value of equity estimated from the price to book multiple of comparable companies.

When $RI_2 = h \cdot RI_1$ this valuation becomes $V_0 / B_0 = m + \frac{x_1^e / B_0 - m \cdot r_e}{1 + r_e - h}$. In this case V_0 / B_0 exceeds m, if return on equity x_1^e / B_0 exceeds $m \cdot r_e$.

3.6.2 When the anchor is proportional to net operating assets

When $y_{i,1} = NOA_0 \cdot m$ and $y_k = y_j = OI$, the GRIV of equation 3 explain the EV to net operating assets multiple as

$$V_0^f = NOA_0 \cdot m + \frac{OI_1 - NOA_0 \cdot m \cdot r_f}{1 + r_f} + \frac{OI_2 - (NOA_0 \cdot m + OI_1 - (C_1 - I_1)) \cdot r_f}{(1 + r_f)(1 + r_f - h)}.$$

When $m = 1$ we recover a version of classic Residual Operating Income Valuation.

Other GRIV that explain the EV to net operating assets multiple need not forecast operating income. For example we can forecast operating cash flows and still anchor the enterprise valuation on a multiple of net operating assets, i.e.

$$V_0^f = NOA_0 \cdot m + \frac{C_1 - NOA_0 \cdot m \cdot r_f}{1 + r_f} + \frac{C_2 - (NOA_0 \cdot m + I_1) \cdot r_f}{(1 + r_f)(1 + r_f - h)}.$$

This valuation discounts a kind of "residual operating cash flows", namely $(C_1 - NOA_0 \cdot m \cdot r_f)$ and $(C_2 - (NOA_0 \cdot m + I_1) \cdot r_f)$, as opposed to residual operating income, and highlights the link between forecasts of operating cash flows and capital expenditures, the current level of V_0^f / NOA_0 and the level of the same multiple of comparable firms as reflected in m. This shows that to explain V_0^f / NOA_0 we need not use classic Residual Operating Income Valuation. Rather we can rely on operating cash flow forecasts, which some analysts prefer because they are insensitive to accruals. When expected future "residual operating cash flows" are positive, $V_0^f / NOA_0 > m$. This type of valuation can reconcile the fact that analyst do refer to the unlevered price to book multiple, and that they do forecast operating cash flows in valuations. There need not be any contradiction in this double practice.

3.6.3 Forecasting operating income and anchoring on another variable

GRIV that forecast operating income, so that $y_{j,1} = OI_1$ and $y_{k,2} = OI_2$, need not anchor the valuation on operating income or on net operating assets, and yet are still equivalent to discouting free cash flows insofar as $z = C - I$.

For example, we can forecast operating income and anchor the valuation on sales so that $y_{i,1} = S_1$, in order to explain the EV to sales multiple. We can forecast operating income and anchor the valuation on EBITDA so that $y_{i,1} = EBITDA_1$, in order to explain the EV to EBITDA multiple. We can forecast operating income and anchor the valuation on free cash flow so that $y_{i,1} = C_1 - I_1$, in order to explain the EV to free cash flow multiple.

Similarly we can forecast operating income and anchor the enterprise valuation on a multiple of sales or of EBITDA, since the EV to sales and EV to EBITDA multiples are popular. This flexibility seems of interest, because, even if the sales multiples and EBITDA multiples are popular, sales and EBITDA have been criticised as measures of value added, as they overlook expenses. Sales overlook all operating expenses and EBITDA overlooks depreciation and amortisation expenses. Instead operating income does take operating expenses into account, and thus often seems more convincing to forecast than sales or EBITDA for valuation purposes (Penman (2012).

Similarly we could forecast operating income and anchor the enterprise valuation on a multiple of free cash flows, as the former may be easier to forecast and a more accurate measure of value added than the latter.

Again the point in this section is that analysts do use valuation multiples and also fundamental valuations, and this double practice may seem contradictory. The valuations we have just reviewed can reconcile this double practice. The contradiction may well not be a real one.

3.6.4 Forecasting free cash flow and anchoring on another variable

Also GRIV that forecast cash flows need not anchor the valuation on cash flows. For example the GRIV in equation 3 can forecast free cash flow, so that $y_{j,1} = C_1 - I_1$ and $y_{k,2} = C_2 - I_2$, and anchor the enterprise valuation on a multiple of sales, i.e.

$$V_0^f = S_1 \cdot m + \frac{C_1 - I_1 - S_1 \cdot m \cdot r_f}{1 + r_f} + \frac{C_2 - I_2 - S_1 \cdot m \cdot r_f}{(1 + r_f)(1 + r_f - h)}.$$

This GRIV highlights how V_0/S_1 is driven by future free cash flow and can reconcile an EV estimate based on the EV to sales multiple of comparable companies, which is the anchor, with discounted free cash flow valuation. Again this seems of interest since free cash flow seems a better measure of value added than sales, yet sales multiples are popular.

3.7 Abnormal Dividend Valuation

We now consider the following special case of GRIV as per equation 3:

- $V_t = V_t^e$, $r = r_e$, $z_1 = d_1$, $y_{j,1} = d_1$ and $y_{k,t} = d_t$ for $t \geq 2$;

- $RI_{k,1} = d_1 - m \cdot y_{i,1} \cdot r_e$ and $RI_{k,2} = d_2 - m \cdot y_{i,1} \cdot r_e$ can be regarded as "abnormal dividends";

- $y_{i,1}$ need not equal d_1 or d_0; when $y_{i,1} = 0$ we obtain classic discounted dividends valuation under the assumption of constant growth of dividends.

When $y_{i,1} = d_1$ this valuation can explain the V_0^e/d_1 multiple, but this multiple is difficult to interpret when d_1 is negative. d_1 may be negative when it measures the net expected payments between the firm and its equity holders during the period $[0, 1]$. To highlight this we introduce the following cash flow identity for all t

$$d_t = d_t^* - c_t^* \quad (5)$$

where:

- d_t^* are the "pure" dividend payments from the enterprise to the equity holders during $[t-1, t]$;

- c_t^* are net capital contributions, i.e. contributions minus distributions, by equity holders to the enterprise during $[t-1, t]$; distributions include share buybacks and other restitutions of equity capital to equity holders.

Notice that $d_1^* \geq 0$, while d_1 can be negative. d_1^* may also be more predictable than d_1, because c_1^* is often not easily predictable. For these reasons we may prefer a valuation model to explain the V_0^e/d_1^* multiple rather than V_0^e/d_1. For the same reasons we may prefer a valuation model that separately forecasts d^* and c^* to a valuation that forecasts d.

When cash flow identity 5 holds, the following special case of equation 3 can explain the $V_0^e/y_{i,1}$ multiple with separate forecasts of d^* and c^*:

- $V_t = V_t^e$, $r = r_e$, $z_1 = d_1$, $y_{j,1} = d_1^*$ and $y_{k,t} = d_t^*$ for $t \geq 2$.

Under these assumptions equation 3 becomes

$$V_0^e = y_{i,1} \cdot m + \frac{d_1^* - y_{i,1} \cdot m \cdot r_e}{1 + r_e} + \frac{d_2^* - (y_{i,1} \cdot m + c_1^*) \cdot r_e}{(1 + r_e)(1 + r_e - h)}.$$

Now "abnormal dividends" are

$$\begin{aligned} RI_{j,1} &= d_1^* - y_{i,1} \cdot m \cdot r_e \\ RI_{k,t} &= d_t^* - r_e \cdot \left(y_{i,1} \cdot m + \textstyle\sum_{i=1}^{t-1} c_i^* \right) \text{ for } t \geq 2. \end{aligned}$$

This valuation can reconcile the equity value estimate $y_{i,1} \cdot m$ based on the $V_0^e / y_{i,1}$ multiple of comparable companies, with forecasts of "pure" dividends d^* that which can explain $V_0^e - y_{i,1} \cdot m$, which is the difference between fundamental value and the said equity value estimate based on the multiples of comparable companies. When $y_{i,1} = d_1^*$ this valuation can naturally explain the V_0^e / d_1^* multiple.

We next focus on the special case where $y_{i,1} = 0$ and cash flow identity 5 holds.

3.7.1 Abnormal dividend growth valuation (ADGV)

We now drop the assumption that $RI_{k,t+1} = h \cdot RI_{k,t}$ for $t \geq 2$ and assume that $y_{i,1} = 0$. Then the "abnormal dividends" now are

$$\begin{aligned} RI_{j,1} &= d_1^* \\ RI_{k,t} &= d_t^* - r_e \cdot \textstyle\sum_{i=1}^{t-1} c_i^* \text{ for } t \geq 2. \end{aligned}$$

Then we can write the general version of ADGV under these assumptions as

$$V_0^e = \frac{d_1^*}{1 + r_e} + \sum_{t=2}^{T} \frac{d_t^* - r_e \cdot \sum_{i=1}^{t-1} c_i^*}{(1 + r_e)^t} + \frac{V_T^e - \sum_{i=1}^{T} c_i^*}{(1 + r_e)^T} \tag{6}$$

$$V_T^e - \sum_{i=1}^{T} c_i^* = \frac{d_{T+1}^*}{1+r_e} + \sum\nolimits_{t=T+2}^{\infty} \frac{d_t^* - r_e \cdot \sum_{i=T+1}^{t-1} c_i^*}{(1+r_e)^{t-T}}$$
$$\lim_{T\to\infty} \frac{V_T^e - \sum_{i=1}^{T} c_i^*}{(1+r_e)^T} \to 0.$$

Equation 6 is a way to re-write discounted dividends valuation under the assumption of cash flow identity 5. The term $V_T^e - \sum_{i=1}^{T} c_i^*$ in equation 6 is the "continuation value", while $\sum_{i=1}^{T} c_i^*$ is the sum of all net capital contributions by equity holders during the period $[0, T]$. Assuming an infinite forecast horizon, i.e. $T \to \infty$, ADGV becomes

$$V_0^e = \frac{d_1^*}{r_e} + \frac{1}{r_e} \sum\nolimits_{t=1}^{\infty} \frac{d_{t+1}^* - d_t^* - r_e \cdot c_t^*}{(1+r_e)^t}.$$

This equation resembles the AEG valuation of Juettner-Nauroth and Ohlson (2005), according to which

$$V_0^e = \frac{x_1^e}{r_e} + \frac{1}{r_e} \cdot \sum\nolimits_{t=1}^{\infty} \frac{x_{t+1}^e - x_t^e - r_e \cdot (x_t^e - d_t)}{(1+r_e)^t}.$$

Therefore ADGV differs from AEG valuation in that d_t^* takes the place of x_t^e and c_t^* takes the place of $(x_t^e - d_t)$ for all t. However ADGV and AEG valuation are equivalent if cash flow identity 5 holds. While AEG valuation focuses on abnormal earnings growth, i.e. $\left(x_{t+1}^e - x_t^e - r_e \cdot (x_t^e - d_t)\right)$, ADGV focuses on abnormal dividends growth, i.e. $\left(d_{t+1}^* - d_t^* - r_e \cdot c_t^*\right)$. Dividends growth is abnormal when the change in dividends $d_{t+1}^* - d_t^*$ differs from the required change in dividends, the latter being the cost of capital r_e multiplied by equity holders' net capital contribution c_t^*. The ADGV formula implies that net capital contributions measured by c_t^* do not affect V_0^e provided they are 0-net-present-value transactions. One such case is when $d_{t+1}^* - c_t^* \cdot r_e = d_t^*$ for all t, and in this case

ADGV becomes

$$V_0^e = \sum_{t=1}^{\infty} \frac{d_t^*}{(1+r_e)^t} - \sum_{t=2}^{\infty} \frac{r_e \cdot \sum_{i=1}^{t-1} c_i^*}{(1+r_e)^t} = \frac{d_1^*}{r_e}.$$

As ADV, also ADGV can explains the "pure" dividend multiple V_0^e/d_1^* as opposed to the dividend multiple V_0^e/d_1^*, but seems less general than ADV, because its valuation anchor can only be $\frac{d_1^*}{r_e}$.

3.8 Abnormal Operating Cash Flow Valuation (ACV)

We now consider the following special case of equation 3:

- $V_t = V_t^f$, $r = r_f$, $z_1 = C_1 - I_1$, $y_{j,1} = C_1$ and $y_{k,t} = C_t$ for $t \geq 2$;

- $RI_{k,1} = C_1 - y_{i,1} m \cdot r_f$ and $RI_{k,2} = C_2 - (I_1 + y_{i,1} m) \cdot r_f$ can be regarded as "abnormal operating cash flows";

- $y_{i,1}$ need not equal C_1 or C_0 and may be 0.

Under these assumptions equation 3 becomes

$$V_0^f = y_{i,1} \cdot m + \frac{C_1 - y_{i,1} \cdot m \cdot r_f}{1 + r_f} + \frac{C_2 - (y_{i,1} \cdot m + I_1) \cdot r_f}{(1 + r_f)(1 + r_f - h)}.$$

"Abnormal operating cash flows" for $t \geq 2$ are

$$RI_{k,t} = C_t - r_f \cdot \left(y_{i,1} \cdot m + \sum_{i=1}^{t-1} I_i \right).$$

ACV can reconcile the EV estimate $y_{i,1} \cdot m$ based on the $V_0^f / y_{i,1}$ multiple of comparable companies, which is the anchor, with forecasts of operating cash flows and capital expenditures of the company in question, which can explain $V_0^f - y_{i,1} \cdot m$. When $y_{i,1} = C_1$ this valuation can explain $V_0^f - C_1 \cdot m$ in a natural way. We next consider the special case whereby $y_{i,1} = 0$, which yields Abnormal Operating Cash flow Growth valuation (ACGV).

3.8.1 Abnormal Operating Cash Flow Growth Valuation (ACGV)

ACGV can explain the EV to (forward) operating cash flow multiple V_0^f / C_1. This multiple tends to be more popular than the EV to (forward) free cash flow multiple. One reason is that C is more likely to be positive than is $C - I$, especially for companies with considerable capital expenditure I. Note that operating cash flow C differs from EBITDA, as explained later. ACGV is a re-writing of discounted free cash flow (DCF) valuation of the enterprise.

To present ACGV we keep the same assumptions as in ACV, except that we now drop the assumption that $RI_{k,t+1} = h \cdot RI_{k,t}$ for $t \geq 2$ and assume that $y_{i,1} = 0$. Then the "abnormal operating cash flows" now are

$$\begin{aligned} RI_{j,1} &= C_1 \\ RI_{k,t} &= C_t - r_f \cdot \textstyle\sum_{i=1}^{t-1} I_i \text{ for } t \geq 2. \end{aligned}$$

Then we can re-write discounted free cash flow valuation to obtain the general version of ACGV under these assumptions

$$\begin{aligned} V_0^f &= \textstyle\sum_{t=1}^{T} \dfrac{C_t - I_t}{(1+r_f)^t} + \dfrac{V_T^f}{(1+r_f)^T} \\ &= \dfrac{C_1}{1+r_f} + \textstyle\sum_{t=2}^{T} \dfrac{C_t - r_f \cdot \sum_{i=1}^{t-1} I_i}{(1+r_f)^t} + \dfrac{V_T^f - \sum_{i=1}^{T} I_i}{(1+r_f)^T} \\ &= \dfrac{C_1}{1+r_f} + \textstyle\sum_{t=2}^{\infty} \dfrac{C_t - r_f \cdot \sum_{i=1}^{t-1} I_i}{(1+r_f)^t}. \end{aligned}$$

The last line is the general version of ACGV valuation. ACGV is the same as residual operating income valuation (ROIV) if C takes the place of OI and

$NOA_0 = 0$. ACGV highlights that operating cash flow C_t to be produced by the enterprise adds value only after it has rewarded, according to the cost of capital r_f, the cumulated future net cash investment in fixed operating assets as measured by the term $\sum_{i=1}^{t-1} I_i$. I_i is the net cash investment in fixed operating assets (i.e. capital expenditure) during the period $[i-1, i]$. ACGV only requires separating the operating cash flow C_t from the capital expenditure cash flow I_t. This separation seems useful for forecasting purposes. Future I reflects expected operating investments/divestments and drive future operating cash flows C.

Assuming an infinite forecast horizon, ACGV can be re-written as

$$V_0^f = \frac{C_1}{r_f} + \frac{1}{r_f} \cdot \sum_{t=1}^{\infty} \frac{C_{t+1} - C_t - r_f \cdot I_t}{(1+r_f)^t} \tag{7}$$

where $C_{t+1} - C_t - r_f \cdot I_t$ is abnormal operating cash flow growth (ACG). ACG is the difference between the change in operating cash flow $C_{t+1} - C_t$ and the required change in operating cash flow $r_f \cdot I_t$, the latter being the cost of capital multiplied by the operating investment of the previous period I_t. Equation 7 resembles the abnormal operating income growth (AOIG) valuation formula of Ohlson and Gao (2006), i.e.

$$V_0^f = \frac{OI_1}{r_f} + \frac{1}{r_f} \cdot \sum_{t=1}^{\infty} \frac{AOIG_{t+1}}{(1+r_f)^t}$$
$$AOIG_{t+2} = OI_{t+2} - OI_{t+1} - r_f \cdot (OI_{t+1} - (C_{t+1} - I_{t+1})).$$

Therefore ACGV and AOIG valuation differ in that C takes the place of OI and I takes the place of $(OI - (C - I))$ for all $t > 0$. While AOIG valuation focuses on abnormal operating income growth, ACGV focuses on abnormal operating cash flow growth.

C can be interpreted as EBITDA only when there is no change in net operating assets not due to capital expenditure. Thus C in general does not coincide with EBITDA. Yet ACGV in equation 7 seems similar in spirit to

$$V_0^f = \frac{EBITDA_1}{r_f} + \frac{1}{r_f} \cdot \sum_{t=1}^{\infty} \frac{v_{t+1}}{(1+r_f)^t}$$
$$v_{t+2} = EBITDA_{t+2} - EBITDA_{t+1} - r_f \cdot (EBITDA_{t+1} - (C_{t+1} - I_{t+1})).$$

Here v_{t+2} could be named "abnormal EBITDA growth". This last valuation is GAEGV when it anchors on EBITDA and $m = 1/r_f$, is a special case of the general valuation framework proposed by Lai (2020), and explains of the popular EV to EBITDA multiple. Instead ACGV explains the popular EV to operating cash flow multiple. Both valuations are equivalent to discounted free cash flow valuation.

4 Conclusion

OJ (2016) extend the AEG valuation by Ohlson and Juettner-Nauroth (2005) and Ohlson (2005) to explain the ubiquitous price to earnings multiple. Lai (2020) adapts the valuation in OJ (2016) to explain other popular valuation multiples. For brevity we refer to these valuations as GAEGV. In a similar spirit this paper has presented generalised residual income valuations (GRIV).

Both GRIV and GAEGV anchor the valuation on some popular valuation multiple. While GAEGV rely on forecasts of a generalised version of abnormal earnings growth (AEG), GRIV rely on forecasts of a generalised version of residual income. GRIV also encompass hybrid valuation models, as they can explain a valuation multiple by forecasting and discounting accounting variables that do not define the valuation multiple. For example GRIV can explain a valuation multiple such as enterprise value to sales, through forecasts of operating cash flows or of operating income or of EBITDA. Similarly GRIV can explain a valuation multiple such as enterprise value to net operating assets, through forecasts of operating cash flows or of EBITDA.

GRIV also provide new flexibility to explain cash flow based valuation multiples through forecasts of cash flows. GRIV can explain price to dividends through abnormal growth in dividends, and EV to operating cash flow through forecasts of abnormal growth in operating cash flows.

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