The Pre-tax *WACC*: Not Pre-tax and Not Useful

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Abstract

Contrary to the claims in the academic finance literature and modern corporate finance textbooks, the pre-tax *WACC* is not a pre-tax rate. Rather, it is the weighted average of the required after-tax rates of return on the firm's operating and interest tax-shield assets and it is useful only for valuing the sum of the after-tax cash flows generated by those assets. In all interesting scenarios, the pre-tax *WACC* varies with the firm's leverage ratio and tax rate, and it is less than the required rate of return on the firm's operating assets. Moreover, the capital cash flow method of capital budgeting that employs the pre-tax *WACC* is never the preferred approach to firm or project valuation.

JEL classification: G12; G31; G32

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1. INTRODUCTION

Harris and Pringle (1985) and Ruback (2002) propose valuing the firm as the present value of its capital cash flows or CCF, the sum of its expected periodic free cash flows (hereafter FCF) and interest tax-shield cash flows, discounted at the rate now commonly referred to as the pre-tax $WACC$.[1] When the firm's periodic FCF are expectationally a regular perpetuity in the amount of $\overline{FCF}$, the firm maintains a fixed leverage ratio, L, pays taxes at the rate τ_C, and is expected to pay interest on its debt at the rate k_D, their approach values the levered firm, as

$$V_0^L = \frac{\overline{FCF} + k_D \tau_C L V_0^L}{\text{Pre-tax } WACC} = \frac{\overline{CCF}}{\text{Pre-tax } WACC}, \tag{1}$$

where

$$\text{Pre-tax } WACC = WACC + \tau_C k_D L \tag{2}$$

$$= k_S (1 - L) + k_D L. \tag{3}$$

$WACC$ in equation (2) is the firm's traditional weighted-average cost of capital and k_S in equation (3) is the after-tax required rate of return on its levered equity. Both Harris and Pringle (1985) and Ruback (2002) claim that the pre-tax $WACC$ is the required rate of return on the levered firm's operating assets, its interest tax-shield asset, and the equity of the otherwise identical unlevered firm. They also assert that the pre-tax $WACC$ is invariant to both the firm's tax rate and its leverage ratio. Modern corporate finance texts (e.g., Berk

[1] Harris and Pringle (1985) call this rate k_0 while Ruback (2002) call it the pre-tax $WACC$, the term now commonly used in the literature to refer to it. I use the term pre-tax $WACC$ in the following when discussing Harris and Pringle's (1985) and Ruback's (2002) specific claims and conclusions.

and DeMarzo (2024) and Brealey, Myers, Allen, and Edmans (2023)) now echo these claims.[2]

Equation (1) does correctly value the levered firm in the scenario described above. However, all of Harris and Pringle's (1985), Ruback's (2002), Berk and DeMarzo's (2024), and Brealey et al.'s (2023) other claims with respect to the pre-tax $WACC$ are false except when the firm also is assumed to pay a fixed proportion of its just-realized operating cash flows as interest at the end of every period. That additional assumption, in turn, necessitates that the required rates of return on the firm's levered equity, its par-valued debt, and its interest tax-shield asset all equal the required rate of return on its operating assets—the scenario Myers (1974, p. 11) termed "uninteresting" more than 50 years ago.

I show that in all scenarios the pre-tax $WACC$ is the weighted-average after-tax cost of or required rate of return on the levered firm's component operating and interest tax-shield assets, hereafter the firm's weighted-average cost of assets or $WACA$. First identified by Ezzell and Miles (1983, p. 30), $WACA$'s only appropriate use is in valuing the sum of the after-tax cash flows generated by those two assets. Moreover, $WACA$ is less than the required rate of return on the firm's operating assets when the required rate of return on the interest-only component of the firm's debt is less than the required rate of return on its operating assets. Like it's $WACC$, the firm's $WACA$ changes when either the firm's tax rate or leverage ratio changes.

[2] Berk and DeMarzo (2024, p. 530) contend that "even in the presence of taxes, a firm's target leverage ratio does not affect the firm's pre-tax $WACC$, which equals its unlevered cost of capital and depends only on the risk of the firm's assets." Berk and DeMarzo (2024) also assert that the unlevered firm, the levered firm's tax-shield asset and the levered firm are correctly valued by discounting the firm's after-tax operating and tax-shield cash flows at the pre-tax $WACC$. Similarly, Brealey et al. (2023, pp. 532–534) assert that the pre-tax $WACC$ "equals the company cost of capital and is independent of leverage."

2. FIRM AND PROJECT VALUATION

2.1. The Cash-Flow Risk Resolution Framework

Bierman and Smidt's (1993) state preference framework is useful for illustrating how the evolution of the firm's operating cash flows and the aspects of capital structure managed (i.e., leverage ratio, debt amount, nature of interest) jointly determine the levered firm's valuation relation and its corollary relations for k_S, $WACC$, and $WACA$.

In the state preference framework, P_s is the beginning-of-period "state price" or present value of a pure security paying \$1 in end-of-period State s and \$0 in all other outcome states. P_s is computed as

$$P_s = \frac{prob_s}{1+k_F}\left[1-\left(\frac{\bar{k}_M-k_F}{\sigma_M^2}\right)\left(k_{M,s}-\bar{k}_M\right)\right], \tag{4}$$

where $prob_s$ is the probability State s is realized, $\bar{k}_M$ is the expected return to the market portfolio of all risky assets (i.e., the "market portfolio" or "the market"), $k_{M,s}$ is the return to the market portfolio in State s, σ_M^2 is the variance of the return to the market portfolio, and k_F is the risk-free rate of interest. The beginning-of-period present value of a distribution of end-of-period cash flows is the sum of the products of each end-of-period cash flow and its state price, a calculation equivalent to discounting the expected end-of-period cash flow at its $CAPM$ required rate of return. I assume throughout that the market return, $k_{M,s}$, will be 45.0% if State u is realized, 20.0% if State m is realized, and -35.0% if State d is realized. The resulting expected market rate of return is 10.0% in both periods. I also assume that the risk-free rate of interest, k_F, is 5.0%. Using equation (4), $P_u = 0.2677$, $P_m = 0.3032$, and $P_d = 0.3814$.

Insert **Figure 1** here

Figure 1 depicts the evolution of the firm's operating cash flow from a just-realized

Time-0 level of $X_0 = \$1{,}000$. At Time 1 and Time 2, the firm will realize either State u, State m, or State d, each with equal probability. The firm's Time-t pre-tax operating cash-flows will be 170% of its Time-$t-1$ after-tax operating cash-flows in State u, 80% of its Time-$t-1$ pre-tax operating cash-flows in State m, or 50% of its Time-$t-1$ pre-tax operating cash-flows in State d. For example, the firm's Time-1, State u operating cash flows will be \$1,700 (= \$1,000 × 1.70). If the firm then realizes state d at Time 2 (i.e., State ud), its operating cash flows will be \$850 (= \$1,000 × 1.70 × 0.50). Valuing those time-1 cash flows using their respective state prices yields the time-0 present value of the time-cash flow distribution

$$PV_0 = \$1{,}7800.2677 + \$800.302 + \$500 \times 0.3814 = \$888.12.$$

The evolution of cash flows in **Figure 1** ensures that the Time-0-expectation of each period's pre-tax operating cash flow, $\overline{X}$, is $X_0 = \$1{,}000$, and the Time-$t$ expectation of every period's pre-tax operating cash flow will be the amount of the just-realized pre-tax the operating cash flow. Given PV_0 and X_0, I find that the expected Time-1 cash flow is valued to Time 0 at the rate

$$k_A = \frac{X_0}{PV_0} - 1 = \frac{\$1{,}000}{\$888.12} - 1 = 12.56\%.$$

I use these same procedures to value the cash flow streams and determine the rates of return discussed in the following.

2.2. Fixed Leverage Ratio Scenarios

2.2.1. Assumptions

The evolution of operating cash flows depicted in **Figure 1** is extended into perpetuity. The firm issues par-valued debt at the beginning of each period sufficient to maintain a fixed market-value leverage ratio, L, and is expected to repay the debt principal amount plus interest at the end of the period.

Because the Time-0 expectation of each period's operating cash flow, $\overline{X}$, is the firm's just-realized operating cash flow, the Time-0 expectation of the levered firm's Time-t value, $\overline{V}_t^L$, is V_0^L; the Time-0 expectation of the debt's Time-t value, $\overline{D}_t$, is $D_0 = LV_0^L$; and the Time-0 expectation of the firm's Time-t interest payment is $k_D\overline{D}_t = k_D L V_0^L$. The firm's interest payments are tax deductible and there are no personal taxes. I also assume that there are no agency costs of debt or costs of financial distress.

To focus attention on the valuation of the firm's interest tax shields, I assume that the debt principal and interest obligations can have different systematic risk, perhaps because they have different contractual priority in the event of default. The expected coupon rate of interest and the required rate of return on the firm's debt, k_D, is the weighted average of the required rates of return the firm's interest-only and principal-only obligations,

$$k_D = \frac{k_P PO_0 + k_I IO_0}{PO_0 + IO_0} = \frac{k_P PO_0 + k_I IO_0}{D_0}, \tag{5}$$

where k_P is the required rate of return on the firm's principal-only debt obligation, PO_0, k_I is the required rate of return on the firm's interest-only debt obligation, IO_0, and $D_0 = PO_0 + IO_0$ is the total Time-0 market value of the firm's debt.

PO_0 equals 0 here because the firm is not expected to repay the debt principal. More precisely, the Time-0 expectation is that neither the notional amount nor the market value of the firm's debt will change through time. Rather, the firm is expected to refinance the same notional and market value of its principal-only obligation at every time t into perpetuity. In effect, the firm is never expected to repay the debt principal amount. As a result, the Time-0 value of the expected principal repayment, PO_0, is 0. Thus, the value of the debt is the value of the firm's interest-only debt obligation, IO_0 and the required rate of return on the firm's debt, k_D, must equal the required rate of return on that obligation, k_I. I maintain the distinction between k_D and k_I in **Section 2.2.2**, below, to highlight the role of k_I in determining the required rate of return on the firm's interest-tax-shield asset in more general settings.

2.2.2. Standard Contractual Interest; $k_D \neq k_I$

The contractual debt interest rate is specified as a percentage of the debt's par value and is set such that the rate of interest expected to be paid on the debt equals k_D, the market's required rate of return on the firm's par-valued debt, in every period. The value of the levered firm is the present value of the perpetuities of its expected, after-tax operating and interest tax-shield cash flows, each discounted at its respective required rate of return. Here,

$$V_0^L = \frac{\overline{FCF}}{k_A} + \tau_C k_D L V_0^L \left(\frac{1}{1+k_I}\right)\left(\frac{1+k_A}{k_A}\right) \tag{6}$$

$$= V_0^U + V_0^{TS}, \tag{7}$$

where k_A is the required rate of return on the firm's operating assets, V_0^U is the value of the otherwise identical unlevered firm,

$$V_0^U = \frac{\overline{FCF}}{k_A},$$

and the value of the levered firm's interest tax-shield asset is

$$V_0^{TS} = \tau_C k_D L V_0^L \left(\frac{1}{1+k_I}\right)\left(\frac{1+k_A}{k_A}\right) \tag{8}$$

$$= \frac{\tau_C k_D L V_0^L}{k_{TS}}. \tag{9}$$

k_{TS} in equation (9) is the effective required rate of return on the firm's interest tax-shield asset,

$$k_{TS} = k_A \left(\frac{1+k_I}{1+k_A}\right). \tag{10}$$

I show in **Appendix A** that the beta of the interest tax-shield asset in equation (9) is

$$\beta_{TS} = \frac{\beta_A + \beta_I k_A}{1+k_A},$$

the weighted average of β_A, the beta of the firm's operating assets, and β_I, the beta of the firm's interest-only debt obligation. Ruback (2002, p. 98) claims that $\beta_{TS} = \beta_A$. That claim obtains, however, only when $k_I = k_A$ such that the firm's interest-only obligation has the same systematic risk as its operating assets and $\beta_I = \beta_A$. When the firm's debt interest payments are systematically risk free and $\beta_I = 0$, the beta of the tax-shield asset is

$$\beta_{TS} = \frac{\beta_A}{1 + k_A}.$$

In all other instances,

$$\frac{\beta_A}{1 + k_A} < \beta_{TS} < \beta_A.$$

Equation (8) values each expected Time-t interest tax shield to Time $t-1$ at the required rate of return on the firm's interest-only liability, k_I, and then present values the resulting perpetuity in advance to Time 0 at k_A, the required rate of return consistent with the evolution of the amount of debt outstanding at Time $t-1$.

Combining equations (6), (8) and (10), the value of the levered firm is

$$V_0^L = \frac{\overline{FCF}}{k_A} + \frac{\tau_C k_D L V_0^L}{k_{TS}}, \tag{11}$$

and using the FCF valuation method, the value of the levered firm is

$$V_0^L = \frac{\overline{FCF}}{k^*}, \tag{12}$$

where

$$k^* = k_A - \tau_C k_D L \left(\frac{1 + k_A}{1 + k_I}\right) \tag{13}$$

is the firm's marginal cost of capital.[3] Substituting $k^* V_0^L = k_A V_0^U = \overline{FCF}$ into the computational formula for the return on equity and simplifying yields

$$k_S = k_A + \left[k_A - k_D - (k_A - k_I)\left(\frac{\tau_C k_D}{1 + k_I}\right)\right]\left(\frac{L}{1 - L}\right). \tag{14}$$

Solving equation (14) for k_A, substituting that result into equation (13), and simplifying, I

[3] Myers (1974) first derived equation (13) in the context of a 1-period project for the case where $k_D = k_I$.

find that the firm's marginal cost of capital also can be written as

$$k^* = k_S(1-L) + k_D(1-\tau_C)L,$$

which is the firm's $WACC$.

Continuing, it follows from equation (11) that the value of the levered firm is the present value of its expected periodic CCF discounted at its $WACA$, where

$$WACA \times V_0^L = k_A V_0^U + k_{TS}\left(\frac{\tau_C k_D L V_0^L}{k_{TS}}\right) \tag{15}$$

$$= k^* V_0^L + \tau_C k_D L V_0^L. \tag{16}$$

Dividing both sides of equation (16) by V_0^L, I find that

$$WACA = k^* + \tau_C k_D L$$

$$= WACC + \tau_C k_D L. \tag{17}$$

Substituting equation (13) for $WACC$ in equation (17) and simplifying yields

$$WACA = k_A - (k_A - k_I)\left(\frac{\tau_C k_D L}{1+k_I}\right). \tag{18}$$

It follows immediately from equation (18) that the beta of the portfolio of the firm's operating and interest tax-shield assets, β_P, is

$$\beta_P = \beta_A - (\beta_A - \beta_I)\left(\frac{\tau_C k_D L}{1+k_I}\right).$$

Equation (17) is the relation for the pre-tax $WACC$ specified by Harris and Pringle (1985) and Ruback (2002). Equation (18) is the functional relation for $WACA$ first identified by Ezzell and Miles (1983, p. 30) for the case where $k_D = k_I$. The equivalence of the firm's pre-tax $WACC$ and its $WACA$ together with equation (1) imply that

$$V_0^L = \frac{\overline{CCF}}{\text{Pre}-\text{tax } WACC} = \frac{\overline{FCF} + \tau_C k_D L V_0^L}{WACA}. \tag{19}$$

However, as noted by Ruback (2002), it is not clear why one would use this approach to value the firm. Equation (19) is unnecessary when D_0 is known and the firm can be valued directly using equation (11) making use of the fact that $D_0 = LV_0^L$. Alternatively, when L is known, solving equation (19) for V_0^L yields equation (12) directly, and equation (19) is again unnecessary.

2.2.3. Standard Contractual Interest; $k_D = k_I$

Miles and Ezzell (1980, 1985) and Ezzell and Miles (1983) consider a scenario where the the expected interest rate and the required rate of return on the firm's par-valued debt in equations (6), (13), (14) and (18), k_D, equals the required rate of return on its interest-only debt obligation, k_I. Miles and Ezzell (1980, 1985) find that the value of the levered firm is

$$V_0^L = \frac{\overline{FCF}}{k_A} + \tau_C k_D L V_0^L \left(\frac{1}{1+k_D}\right)\left(\frac{1+k_A}{k_A}\right) = \frac{\overline{FCF}}{k^*}, \quad (20)$$

[4]

where the firm's marginal cost of capital is

$$\begin{aligned} k^* &= k_A - \tau_C k_D L \left(\frac{1+k_A}{1+k_D}\right) \quad (21) \\ &= k_S (1-L) + k_D (1-\tau_C) L = WACC, \end{aligned}$$

[4] Many writers, including Harris and Pringle (1985), Taggart (1991), Ruback (2002), Cooper and Nyborg (2006), Massari, Roncaglio, and Zanetti (2007), Brealey et al. (2023), Berk and DeMarzo (2024), and even Ezzell and Miles (1983), misconstrue the value of the interest tax shield, $\tau_C k_D L V_0^L (1+k_A) / (k_A (1+k_D))$ in equation (20), claiming that it values the first period's interest tax shield at the return on debt and values all subsequent interest tax shields at the return on assets. It does not. The firm-value relation consistent with these writers' claim is

$$V_0^L = \frac{\overline{FCF}}{k_A} + \tau_C k_D L V_0^L \left(\frac{1}{1+k_D} + \frac{1}{k_A (1+k_A)}\right),$$

and not equation (20).

and that the required rate of return on the levered firm's equity is

$$k_S = k_A + (k_A - k_D)\left(1 - \frac{\tau_C k_D}{1 + k_D}\right)\left(\frac{L}{1 - L}\right). \tag{22}$$

Additionally, Ezzell and Miles (1983, p. 30) find that the firm's weighted-average cost of assets is

$$WACA = k_A - (k_A - k_D)\left(\frac{\tau_C k_D L}{1 + k_D}\right) \tag{23}$$

$$= WACC + \tau_C k_D L. \tag{2}$$

Hereafter, I refer to equations (20) through (23) as the Miles and Ezzell's family of capital structure relations.

Insert **Table 1** here

Table 1, Panel B presents the values of the unlevered firm, the levered firm, and the interest tax shield asset in addition to the various rates of return discussed above assuming the firm's cash flows evolve into perpetuity as illustrated in **Figure 1**. For future reference, note that $k_D = k_I = k_F = 5.0\%$, the value of the levered firm is \$5,874.06 using both the FCF and CCF valuation methods, $k^* = 11.92\%$, $k_S = 17.53\%$, and $WACA = 12.52\%$.

2.2.4. Interest Proportional to Realized Operating Cash Flow

Also employing a scenario consistent with that presented in **Figure 1** continued into perpetuity, Berk and DeMarzo (2024, pp. 668) consider a firm that issues debt comprised entirely of a promise to pay as interest at every time t the proportion λ of its just-realized FCF_t. In this case, the interest payments have the same systematic risk as the firm's operating cash flows in every period and are valued at k_A in every period (i.e., $k_I = k_A$ in every period). Here again, the Time-t expectation of each of the firm's future periodic free cash flow, $\overline{FCF}$, is the just-realized FCF_t, and neither the levered firm's value, the value of its component unlevered firm, its expected interest payment, its expected periodic interest tax shield, nor the value of its debt is expected to change through time. The Time-0 value of the levered firm is

$$V_0^L = \frac{\overline{FCF}}{k_A} + \frac{\tau_C \lambda \overline{FCF}}{k_A}$$
$$= V_0^U + \tau_C \lambda V_0^U = V_0^U \left(1 + \tau_C \lambda\right),$$

and the Time-0 value of the firm's debt is

$$D_0 = \frac{\lambda \overline{FCF}}{k_A}$$
$$= \lambda V_0^U. \tag{24}$$

As noted by Berk and DeMarzo (2024, p. 670), these relations imply that the firm's targeted leverage ratio is

$$L = \frac{D_0}{V_0^L} = \frac{\lambda V_0^U}{V_0^U \left(1 + \tau_C \lambda\right)} = \frac{\lambda}{1 + \tau_C \lambda}, \tag{25}$$

and that

$$\lambda = \frac{L}{1 - \tau_C L},$$

neither of which depends upon time. Thus, Miles and Ezzell's family of capital structure relations, equations (20) through (23), obtain here. Specifically, the value of the firm is

$$V_0^L = \frac{\overline{FCF}}{k_A} + \tau_C k_A L V_0^L \left(\frac{1}{1 + k_A}\right)\left(\frac{1 + k_A}{k_A}\right) \tag{20}$$
$$= \frac{\overline{FCF}}{k_A} + \tau_C L V_0^L$$
$$= \frac{\overline{FCF}}{k_A \left(1 - \tau_C L\right)}, \tag{20'}$$

the firm's marginal cost of capital or $WACC$ is

$$k^* = k_A - \tau_C k_A L \left(\frac{1 + k_A}{1 + k_A}\right) \tag{21}$$

$$= k_A \left(1 - \tau_C L\right), \tag{21'}$$

the required rate of return on levered equity is

$$k_S = k_A + (k_A - k_A)\left(1 - \frac{\tau_C k_A}{1 + k_A}\right)\left(\frac{L}{1 - L}\right) \tag{22}$$

$$= k_A, \tag{22'}$$

and the firm's weighted average cost of assets is

$$WACA = k_A - (k_A - k_A)\left(\frac{\tau_C k_A L}{1 + k_A}\right) \tag{23}$$

$$= k_A. \tag{23'}$$

Here again, $\overline{FCF} = \$700.00$. The firm's interest payments are credit-risk free in this scenario because the firm aways can make the promised interest payment, λFCF_t. However, the firm's debt, the value of which is the present value of those interest payments, has the same systematic risk as the firm's operating assets. Thus, the expected Time-t interest tax shield must be valued to Time t-1 and all preceding times at k_A. In effect, the firm's interest-only obligation must be valued such that

$$\lambda \overline{FCF} = k_A \overline{D} = k_D \overline{D},$$

or such that $k_D = k_A = 12.56\%$.

Table 1, Panel C presents the values of the unlevered firm, the levered firm, and the interest tax shield asset in addition to the various rates of return discussed here assuming the firm's cash flows evolve into perpetuity as illustrated in **Figure 1**. In this scenario, $k_D = k_A = 12.56\%$ as just noted, the value of the levered firm is \$6,333.24 using both the FCF and CCF valuation methods, $k^* = 11.05\%$, $k_S = 12.56\%$, and $WACA = 12.56\%$. The value of the levered firm here greatly exceeds that value of the levered firm computed using Miles and Ezzell's family of capital structure relation and that difference is due entirely to the greater value of the interest tax-shield asset in this scenario.

2.3. Fixed Debt Schedule Scenarios

2.3.1. Assumptions

The evolution of operating cash flows depicted in **Figure 1** is again extended into perpetuity such that the Time-0 expectation of each future Time-t's operating cash flow, $\overline{X}$, equals $X_0 = \$1{,}000.00$, the firm's just-realized operating cash flow. Here also, the firm's interest payments are tax deductible, and there are no personal taxes and no agency costs of debt or costs of financial distress.

2.3.2. Standard Contractual Interest

The assumed evolution of the firm's operating cash flow is consistent with Modigliani and Miller's (1958, 1963) cash-flow-risk assumption.[5] They also assume, however, that the firm issues default-risk-free perpetual debt. These two assumptions are inconsistent. Specifically, the firm cannot issue any fixed amount of perpetual, default-risk-free debt in this scenario because the firm's operating cash flow approaches zero in an infinite number of future states.

If, instead of issuing a fixed amount of risk-free debt, the firm issues a fixed notional amount of risky, fixed-rate debt having a Time-0 par coupon rate of c, then the promised periodic interest payment is $C_t = cD_0$ and the Time-0 expectation of the Time-t interest payment is $\overline{C}_t$. The value of the levered firm again equals the value of the unlevered firm plus the value of the interest tax shield, but Modigliani and Miller's (1963) Proposition I becomes

$$GAPV = V_0^L = \frac{\overline{FCF}}{k_A} + \tau_C \sum_{t=1}^{\infty} \frac{\overline{C}_t}{\prod_{n=1}^{t} \left(1 + {}_{n-1}\bar{k}_{I,n}^{(t)}\right)} \tag{26}$$
$$= V_0^U + V_0^{TS}.$$

[5] Modigliani and Miller (1963) adopt the cash-flow risk resolution assumption detailed in Modigliani and Miller (1958, p. 265, n. 6).

Equation (26) is a generalized version of Myers's (1974) APV, hereafter $GAPV$, where ${}_{n-1}\bar{k}_{I,n}^{(t)}$ is the Time-0 expectation of the market's period-n required rate of return on the firm's Time-t interest tax shield.[6] Equation (26) can be applied in practice, but it provides no path to inferring values of k^*, k_S, k_A, or $WACA$ applicable in all periods.

2.3.3. Interest Proportional to Realized Operating Cash Flow

Finally, consider the case where the firm issues debt at Time 0 having market and par values of D_0. The debt's market value is the present value of the firm's promise to pay the fixed proportion λ of its FCF as interest at the end of every period into perpetuity. Given D_0 and V_0^U, it follows from equation (24) that $\lambda = D_0/V_0^U$. As noted in **Section 2.2.4**, the value of the debt will change every period to maintain the fixed market-value leverage ratio identified in equation (25). All of the results presented in **Section 2.2.4** follow directly. In particular, the capital structure propositions presented in equations (20) through (23) obtain with $k_S = k_D = k_{TS} = k_A$.

2.4. Discussion

Equations (15) through (18) clearly demonstrate that the pre-tax $WACC$ is the weighted-average cost of the firm's assets, $WACA$, the rate appropriately used to value the levered firm as the present value of its CCF's when the firm maintains a fixed market-value leverage ratio. That result neither requires nor implies that the rate of return on the interest tax-shield asset equals the required return on the firm's operating assets. Rationally, the firm's

[6] Perhaps the most universally applicable form of Modigliani and Miller's (1963) Proposition I (or Myers's (1974) APV) is

$$GAPV = V_0^L = \sum_{t=1}^{T} \frac{\overline{FCF}_t}{(1+k_A)^t} + \tau_C \sum_{t=1}^{T} \frac{{}_{t-1}k_{D,t}\overline{D}_{t-1}}{\prod_{n=1}^{t}\left(1 + {}_{n-1}\bar{k}_{I,n}^{(t)}\right)},$$

where ${}_{t-1}k_{D,t}$ is the Time-0 expectation of the period-t required rate of return on the amount of debt expected to be outstanding at the beginning of that period, D_{t-1}.

periodic interest tax-shield asset cannot be systematically riskier than its periodic operating cash flows in the scenario assumed here. Thus, k_I must be less than or equal to k_A, and, as noted by Booth (2007, p. 38), $WACA$ is the required rate of return on operating assets only in the unlikely scenario where $k_{TS} = k_D = k_I = k_A$. Otherwise, $WACA$ is less than k_A and it is decreasing in both the corporate tax rate and the firm's leverage ratio. These relationships are intuitive. Any increase in leverage increases the firm's periodic interest tax shield and must increase V_0^{TS}. V_0^U is not affected by an increase in leverage and k_{TS} is affected by an increase in leverage only if it causes an increase in k_I. In all other circumstances, the weighted-average of the required rates of return on the firm's operating and tax-shield assets must decrease as the firm's leverage increases. Any increase in the firm's tax rate also increases the amount and value of the firm's periodic interest tax shield while decreasing the firm's after-tax operating cash flows and the value of the unlevered firm. The resulting decrease in the value of the unlevered firm together with the increase in the value of the interest tax shield causes $WACA$ to decrease.

Equations (6), (13), (14) and (18) define a family of capital structure relations. Importantly, equations (13), (14) and (18) are derived from equation (6); none of these relations and none of the rates of return included in them is a matter of choice. Both Harris and Pringle (1985) and Ruback (2002) set $k_I = k_A$ in equation (18). However, they ignore the implications of that action for k_D. The firm's debt is expectationally perpetual, its principal has a present value of 0, and, by equation (5), k_D must equal k_I, the rate Harris and Pringle (1985) and Ruback (2002) set equal to k_A.[7] In effect, Harris and Pringle (1985) and

[7]Taggart (1991, p. 12) notes that the sum of the firm's after-tax operating and interest tax-shield cash flows is appropriately valued at k_A when the firm's capital structure is continuously adjusted to L. Notwithstanding Arzac and Glosten's (2005) claim to the contrary, neither Harris and Pringle (1985) nor Ruback (2002) couch their analyses in continuous time. Moreover, both Harris and Pringle (1985) and Ruback (2002) use discretely compounded discount factors to value end-of-period cash flows in their examples.

Ruback (2002) are requiring the firm to pay a fixed proportion of its realized operating cash as interest at the end of every period. Indeed, imposing the requirement that $k_D = k_I = k_A$ throughout equations (6), (13), (14) and (18), yields the family of capital structure relations consistent with Harris and Pringle's (1985) and Ruback's (2002) assertion,

$$V_0^L = \frac{\overline{FCF}}{k_A\left(1 - \tau_C L\right)} \tag{6'}$$

$$k^* = k_A\left(1 - \tau_c L\right), \tag{13'}$$

$$k_S = k_A, \tag{14'}$$

and

$$WACA = k_A, \tag{18'}$$

which is the same family of relations, equation (20′) through equation (23′), implied by Berk and DeMarzo's (2024) analysis as discussed in **Section 2.2.4**.

The assumed operating-cash-flow generating process in **Figure 1** ensures that $\overline{FCF}_t = \overline{FCF} = FCF_0$, $\overline{V}_t^L = V_0^L$ and that $\overline{D}_t = D_0$ for all t. Thus, Harris and Pringle (1985) and Ruback (2002) effectively assume that the firm's debt is a claim to a perpetual stream of interest payments, each of which is a fixed proportion of the firm's realized operating cash flow and subject to the same systematic risk as the firm's operating assets. Consequently, the stockholders and debtholders share the systematic risk of the firm's operating assets in proportion to the value of their respective claims. In effect, the firm's interest payments are tax-deductible dividends. Because the debt and equity claims are isomorphic, both k_S and k_D must equal k_A. Inconsistent with that requirement and the discussion in **Section 2.2.1**, Harris and Pringle (1985) and Ruback (2002) assume in their respective rationalizations of the pre-tax $WACC$ that k_S is greater than k_A and that the firm's par-valued perpetual debt has an expected coupon rate of k_D that is less than k_A.

Ruback's (2002) claim that the pre-tax $WACC$ is the required rate of return on the firm's unlevered equity and interest tax-shield asset follows directly from his assertion that a firm maintaining a fixed leverage ratio will maintain a fixed interest coverage ratio. As shown by Berk and DeMarzo (2024) and illustrated here, a firm maintaining a fixed interest coverage ratio will maintain a fixed leverage ratio. However, Ruback's (2002) claim of the converse is not true. k_A will be greater than the required rate of return on firm's interest-only obligation as long as it has issued ordinary debt and is not expected to default on the entirety of its promised interest payment in every future state. Ruback (2002, p. 85) explicitly assumes that the firm's capital structure consists of "ordinary debt and common equity." Debt that pays interest as a fixed proportion of the firm's realized periodic operating cash flow is not obviously "ordinary," but it is the *sine qua non* of Ruback's (2002) claims.

Lastly, the pre-tax $WACC$ or $WACA$ is not a pre-tax rate. The firm's periodic after-tax operating cash flows are definitionally after-tax while its interest tax shields exist only because it employs leverage and the resulting interest payments are deductible for tax purposes. Both of these cash flows are after-tax quantities and their sum is appropriately valued at an after-tax rate. That rate, $WACA$, is the weighted average of k_A, the required rate of return on the firm's operating assets, and k_{TS}, the required rate of return on the firm's tax-shield asset. k_A is invariant to the existence or magnitude of corporate income taxation. Thus, k_A is an after-tax rate when the firm's interest tax rate is greater than zero. k_{TS} is invariant to the firm's tax rate and its leverage ratio but exists only as a consequence of leverage and taxes. The weighted average of these two rates also must be an after-tax rate. The same conclusion results when $WACA$ is viewed from Harris and Pringle's (1985) and Ruback's (2002) claims-on-cash-flows perspective. Equation (3) defines the pre-tax $WACC$ and $WACA$ as the weighted-average of the required after-tax rate of return on the firm's levered equity and the required pre-tax rate of return return on its debt. These are the rates used by the market to value the equity and debt claims, respectively, to the firm's portfolio of operating and tax-shield assets. Adding $\tau_C k_D L$ to the firm's $WACC$ to obtain its pre-tax $WACC$ does not result in a pre-tax quantity because doing so does not remove from that sum the effect of taxes on the required rate of return on levered equity. However, it does is make clear that equity holders value after-tax cash flows, including the tax benefit of debt, while debtholders value pre-tax cash flows.

3. *WACA*'s Sensitivity to the Tax Rate and Leverage

Table 2 presents the values of $WACC$ and $WACA$ computed using different combinations of $k_D = k_I$, L, and τ_C given that $k_A = 12.56\%$. **Table 2** also presents the corresponding $\overline{CCF}$ and the value of the levered firm obtained by valuing the firm's expected perpetual $\overline{FCF}$ at its $WACC$ or by valuing its expected perpetual $\overline{CCF}$ at its $WACA$.

Insert **Table 2** here

Table 2 presents several important results. First, $WACC$ equals $WACA$ and both equal k_A when there is no tax advantage to debt (i.e., when $\tau_C = 0$ or $L = 0$). Second, $WACC$ is much more sensitive to changes in leverage than is $WACA$. $WACC$ changes with L but the firm's pre-tax operating cash flows do not. Any increase in leverage, however, decreases $WACA$ and increases the firm's $\overline{CCF}$. Thus, the increase in value resulting from an increase in leverage has value-increasing effects on both the numerator and denominator of the CCF valuation relation in equation (19). Third, $WACA$ is less than k_A when there is a tax advantage to debt, and $WACA$ decreases as the tax rate increases. Finally, discounting the firm's perpetual after-tax operating cash flows at $WACC$ and discounting its perpetual $\overline{CCF}$ at $WACA$ value the levered firm identically.

The data in **Table 2** also can be used to demonstrate that separately discounting the firm's after-tax operating cash flows and tax-shield cash flows at $WACA$ correctly values neither the unlevered firm nor the tax-shield asset. Consider the firm having a leverage ratio of 40.0% and paying taxes at a rate of 30.0%. The firm's perpetual after-tax operating cash flows are expected to be \$700.00 and the required rate of return on its operating assets is 12.56%. Thus, the value of the unlevered firm is $V_0^U = \$700.00/0.1256 = \$5{,}573.25$. The firm's interest tax shield is expected to be \$35.24 per period, the required rate of return on the interest tax-shield asset, k_{TS}, is $0.1256 \times 1.05/1.1256 = 11.72\%$, and $V_0^{TS} = \$35.24/0.1172 = \$300.81$. In the spirit of Myers's (1974) APV, the levered firm's $GAPV$

$$V_0^L = V_0^U + V_0^{TS} = \$5{,}573.25 + \$300.81 = \$5{,}874.06,$$

the same value obtained by discounting the firm's $\overline{FCF}$ at its $WACC$,

$$V_0^L = \frac{\overline{FCF}}{WACC} = \frac{\$700.00}{0.1192} = \$5{,}874.06.$$

Valuing the firm's $\overline{CCF}$ at $WACA$, I also find that

$$V_0^L = \frac{\overline{CCF}}{WACA} = \frac{\$735.24}{0.1252} = \$5{,}874.06.$$

In contrast, Harris and Pringle (1985) and Ruback (2002) contend that the levered firm's component unlevered firm and interest tax-shield assets are correctly valued by discounting their respective cash flows at $WACA$. Specifically, they claim that

$$V_0^{U\prime} = \frac{\$700.00}{0.1252} = \$5{,}592.48 > V_0^U \qquad \text{and} \qquad V_0^{TS\prime} = \frac{\$35.24}{0.1252} = \$281.58 < V_0^{TS}.$$

Clearly, Harris and Pringle (1985) and Ruback (2002) correctly value the levered firm that maintains a fixed leverage ratio. However, they over-value the component unlevered firm and undervalue the component interest tax-shield asset.

4. Using $WACA$ to Value a Finite-lived Project

Ruback (2002, pp. 90-92) presents an example comparing the FCF and CCF approaches to valuing a finite-lived capital project assuming that the operating and interest tax-shield cash flows are valued at k_A, the rate Ruback also asserts is the pre-tax $WACC$. He claims to find that both methods result in the same Time-0 project value, but leaves unexplained his finding that they produce different intermediate-date project values. Booth (2007) identifies that and several other issues with Ruback's (2002) example. Rather than rehash those issues, I illustrate the valuation of a finite-lived project using $WACA$ when the project is funded according to a fixed debt schedule rather than at a fixed leverage ratio.

Assume that **Figure 1** illustrates the known distribution of operating cash flows for a two-period project that is initially funded in part with \$300 of debt. The debt requires repayment of \$125 of principal plus interest on the entire \$300 at Time 1, and repayment of the remaining \$175 of principal with interest at Time 2. The debt is issued at par and reprices to par at Time 1.[8] Debt issued in State u or State m at Time 1 will be risk free with a par coupon rate of 5.0% and a required return on the debt's principal and interest of 5.0%. If State d is realized at Time 1, the project will default on its interest payment in State d at Time 2, but will make its promised principal payment in all Time-2 states. The par coupon of the debt issued at Time 1 in State d will be 8.34% and the required return on the resulting expected Time-2 interest tax-shield asset will be 8.92%.[9] The Time-0 expectation of the Time-1 coupon rate and the required rate of return on the debt maturing at Time 2 is the weighted average of the required rates of return on the debt principal and interest, 5.19%. The project's expected free cash flow in each period (i.e., its $\overline{FCF}_t$) is \$700.00, the Time-0 expectation of its Time-2 interest tax shield is \$2.72, and $\overline{CCF}_2$, is \$702.72.

I assume that equation (14) has been used to infer the value of k_A from the observed values of k_S, k_I, k_D, τ_C, and L of a pure-play firm having the same systematic risk as the

[8]This assumption simplifies the example by ensuring that the Time-1 value of the risky expected Time-2 interest tax shield is discounted to Time 0 at the risk-free rate.

[9]The Time-2, State-dd coupon rate of interest, c_{dd}, is the solution to the equation

$$\$175.00 = \$175.00 \times 0.2677\,(1 + c_{dd}) + \$175.00 \times 0.3032\,(1 + c_{dd}) + \$175.00 \times 0.3814.$$

The required rate of return on the expected Time-2 interest tax-shield asset, $k_{TS,2}$, is

$$\begin{aligned} k_{TS,2} &= \frac{(6 \times \$175.00 \times 0.05 + 2 \times \$175.00 \times 0.0834)\,/9}{(2 \times \$175.00 \times 0.05 \times 0.9524 + \$175.00 \times 0.0834 \times (0.2677 + 0.3032))\,/3} - 1 \\ &= 8.92\%, \end{aligned}$$

where $\$175.00 \times 0.05$ is the Time-2 interest payment in all states if State u or State m is realized at Time 1, and $\$175.00 \times 0.0834$ is the Time-2 interest payment in State u and State m if State d is realized at Time 1.

project and found to be 12.56%. The project has different $WACA$s for Period 1 and Period 2 because the firm's leverage ratio changes at time 1. The process of finding ${}_1WACA_2$, the discount rate at which to value the Time-2 $\overline{CCF}$ to Time 1, begins with the relation for the Time-0 expectation of the levered project's Time-1 value,

$$V_1^L = \frac{\overline{FCF}_2}{1+k_A} + \frac{\tau_C k_{D,2} D_1}{1+k_{I,2}} \tag{27}$$

$$= V_1^U + V_1^{TS} \tag{28}$$

where $k_{D,2} = k_{I,2} = 5.19\%$ is the required return on the tax-shield asset in period 2, and V_1^U and V_1^{TS} are the Time-0 expectation of the Time-1 value of the unlevered project and interest tax-shield asset, respectively.[10] As shown by Ezzell and Miles (1983, p. 29), equation (28) implies that

$$\begin{aligned} 1 + {}_1WACA_2 &= (1+k_A)\left(\frac{V_1^U}{V_1^L}\right) + (1+k_{I,2})\left(\frac{V_1^{TS}}{V_1^L}\right) \\ &= 1 + k_A - (k_A - k_{I,2})\left(\frac{V_1^{TS}}{V_1^L}\right) \\ &= 1 + k_A - (k_A - k_{I,2})\left(\frac{V_1^{TS}\left(1 + {}_1WACA_2\right)}{\overline{CCF}_2}\right) \\ &= \frac{(1+k_A)\,\overline{CCF}_2}{\overline{CCF}_2 + (k_A - k_{I,2})\,V_1^{TS}}. \end{aligned}$$

Here, ${}_1WACA_2 = 12.53\%$, and the Time-0 expectation of the project's Time-1 value is

$$V_1^L = \frac{\overline{CCF}_2}{1 + {}_1WACA_2} = \frac{\$702.72}{1.1253} = \$624.48.$$

The value of the firm computed using the FCF method must equal the value computed using both the CCF and $GAPV$ methods at every time t. Thus,

$$V_t^L = \frac{\overline{FCF}_{t+1} + V_{t+1}^L}{1 + {}_tWACC_{t+1}} = \frac{\overline{CCF}_{t+1} + V_{t+1}^L}{1 + {}_tWACA_{t+1}}. \tag{29}$$

[10] Although V_1^L is easily computed directly from equation (27), the task here is to value the project's CCF using the appropriate period-specific $WACA$s.

Rearranging equation (29), I find that

$$1 + {}_t WACC_{t+1} = \frac{(1 + {}_t WACA_{t+1})}{1 + \dfrac{\tau_C k_{D,t} D_{t+1}}{\overline{FCF}_{t+1} + V^L_{t+1}}}.^{11} \tag{30}$$

Substituting the known quantities into equation (30) and solving, I find ${}_1WACC_2 = 12.09\%$, and the Time-1 value of the project is

$$V^L_1 = \frac{\$700.00}{1.1209} = \$624.48,$$

the same value obtained above. Substituting the known quantities directly into the relevant $GAPV$ valuation relation, equation (27), also yields $V^L_0 = \$624.88$.

Continuing, the project's Time-0 $GAPV$ is given by

$$\begin{aligned} V^L_0 &= \frac{\overline{FCF}_1 + V^U_1}{1 + k_A} + \frac{\tau_C k_{D,1} D_0 + V^{TS}_1}{1 + k_{I,1}} \\ &= V^U_0 + V^{TS}_0, \end{aligned} \tag{31}$$

which implies that[12]

$$1 + {}_0 WACA_1 = 1 + k_A - (k_A - k_{I,1}) \left(\frac{V^{TS}_0}{V^L_0} \right).$$

Proceeding as before, I find

$$1 + {}_0 WACA_1 = \frac{(1 + k_A) \left(\overline{CCF}_1 + \tau_C k_{D,1} D_0 + V^L_1 \right)}{\overline{CCF}_1 + \tau_C k_{D,1} D_0 + V^L_1 + (k_A - k_{I,1}) V^{TS}_0}. \tag{32}$$

[11] The relationship between $WACC$ and $WACA$ in equation (30) differs from that in Booth's (2007, p. 42) equation (32). Apparently efforting to enhance comparability between his and Ruback's (2002) results, Booth assumes that $WACA$ (K_U in his notation) is unaffected by a change in the firm's leverage ratio.

[12] The Time-1 debt was priced to par such that the Time-1 value of the expected Time-2 interest tax shield is the same regardless of the state realized at Time 1, and thus is expectationally risk free. The single-period debt issued at Time 0 is risk free by assumption. Thus, $k_{D,1} = k_{I,1} = k_F$ is the appropriate rate at which to value both $\tau_C k_{D,1} D_0$ and V^{TS}_1 to Time 0. Otherwise, these two Time-1 quantities would be valued to Time 0 at different rates thereby further complicating the calculation of ${}_0 WACA_1$.

The Time-1 interest tax shield is $\$300.00 \times 0.05 \times 0.30 = \$4.50$ with a present value of $\$4.50/1.05 = \$4.29$. Additionally, $k_{D,1} = k_{I,1} = k_F = 5.0\%$, and all other quantities are as specified above. Substituting those quantities into equation (32), I find ${}_0WACA_1 = 12.52\%$, and the project's Time-0 value is \$1,181.14.

Using a similar approach to estimating the $WACA$s needed to separately discount $\overline{CCF}_1$ and V_1^L to Time 0, I find that $\overline{CCF}_1$, \$704.50, has a Time 0 value of \$626.18 when discounted at its $WACA$ of 12.51%, and ${}_0V_1^L$, \$624.48, has a Time-0 value of \$554.96 when discounted at its $WACA$ of 12.52%. This method also values the project at \$1,181.14. To value the project's FCF to time 0 requires an estimate of ${}_0WACC_1$. Using equation (30), I find that ${}_0WACC_1$ is 12.14%. Discounting the sum of V_1^L and $\overline{FCF}_1$ at this rate, I again find that the value of the levered firm is \$1,181.14. Using the $GAPV$ relation, equation (31), to compute V_0^L directly, also yields $V_0^L = \$1{,}181.14$.

These results are inconsistent with Ruback's (2002) claim that all of a finite-lived project's CCF are valued at a single $WACA$ even when it is funded according to a fixed debt schedule.As shown here, the $WACA$ used to value $\overline{CCF}_{t+1} + V_{t+1}^L$ to time t will differ from the $WACA$ used to value $\overline{CCF}_{t+2} + V_{t+2}^L$ to time $t+1$ when the project's leverage ratio changes thereby causing the relative values of the firm's operating and tax-shield assets to change. Moreover, a particular risky interest tax shield can require discounting at a different rate in each period as its systematic risk changes. As a result, valuing the project's CCF using one or a series of $WACA$s is more complex, not simpler, than valuing its FCF using one or a series of $WACC$s. Indeed, valuing the project directly using the $GAPV$ method reflected in equations (27) and (31) is much simpler than either the CCF or FCF approaches.

5. SUMMARY AND CONCLUSION

The pre-tax $WACC$ or $WACA$ is not a pre-tax rate as claimed in the literature. Moreover, the pre-tax $WACC$ is not the required rate of return on the firm's operating assets or unlevered equity and it is not invariant to the firm's leverage ratio and tax rate obtain except when the firm pays a fixed proportion of its operating cash flow as interest at the end of every period. Instead, the pre-tax $WACC$ is in all scenarios the weighted-average of the after-tax required rates of return on the firm's operating and interest tax-shield assets, and it is useful only for valuing the stream of periodic, after-tax cash flows expected to be generated by those assets.

Because the firm's pre-tax $WACC$, like its $WACC$, is a function of the firm's leverage ratio and tax rate, the firm can be appropriately valued using a single pre-tax $WACC$ only when it is funded at the same, fixed market-value leverage ratio and taxed at the same rate in every period. Even then, the related CCF method almost certainly will not be the preferred valuation approach as the FCF method is simpler than both the CCF and $GAPV$ methods when the firm is funded at a fixed leverage ratio. When the firm is funded according to a fixed debt schedule such that it's leverage ratio changes over time, the $GAPV$ method is simpler than both the CCF and FCF methods

The evidence presented here supports the conclusion that the pre-tax $WACC$ is a somewhat interesting artifact of no practical importance. Perhaps more importantly, that evidence also shows that the academic literature misconstrues and misrepresents the pre-tax $WACC$ by confounding it with the required rate of return on operating assets.

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Appendix A: Derivation of $\boldsymbol{\beta_{TS}}$

Because both sides of the firm's balance sheet have the same systematic risk, the weighted-average of the betas of the firm's liabilities and equities must equal the weighted-average of the betas of its assets. Thus,

$$\beta_S (1 - L) + \beta_D L = \frac{\beta_A V_0^U + \beta_{TS} V_0^{TS}}{V_0^L}, \tag{A.1}$$

where β_S is the beta of the levered firm's equity, β_D is the beta of the levered firm's debt, β_A is the beta of the levered firm's operating assets, and β_{TS} is the beta of the fim's interest tax-shield asset. Equations (A.1) and (6) and the formula for the beta of the levered firm's equity consistent with equation (14),

$$\beta_S = \beta_A + \left[\beta_A - \beta_D - (\beta_A - \beta_I)\left(\frac{\tau_C k_D}{1 + k_I}\right)\right]\left(\frac{L}{1 - L}\right),$$

together imply that

$$\left[\beta_A + \left(\beta_A - \beta_D - (\beta_A - \beta_I)\left(\frac{\tau_c k_D}{1 + k_I}\right)\right)\left(\frac{L}{1 - L}\right)\right](1 - L) + \beta_D L = \frac{\beta_A \left[1 - \tau_C L\left(\frac{k_D}{1 + k_I}\right)\left(\frac{1 + k_A}{k_A}\right)\right] V_0^L + \beta_{TS}\tau_C L\left(\frac{k_D}{1 + k_I}\right)\left(\frac{1 + k_A}{k_A}\right) V_0^L}{V_0^L},$$

and

$$\beta_{TS}\tau_C L\left(\frac{k_D}{1 + k_I}\right)\left(\frac{1 + k_A}{k_A}\right) = \beta_A \tau_C L\left(\frac{k_D}{1 + k_I}\right)\left(\frac{1 + k_A}{k_A}\right) - (\beta_A - \beta_I)\,\tau_c L\left(\frac{k_D}{1 + k_I}\right), \tag{A.2}$$

where β_I is the beta of the firm's interest-only liability. Simplifying equation (A.2) yields the beta of the interest tax-shield asset,

$$\begin{aligned}\beta_{TS} &= \beta_A - (\beta_A - \beta_I)\left(\frac{k_A}{1 + k_A}\right) \\ &= \frac{\beta_A + \beta_I k_A}{1 + k_A}.\end{aligned}$$

Figure 1. The Evolution of Cash Flows Through Time When Cash-flow Risk Resolves at a Constant Periodic Rate

The firm's Time-t after-tax operating cash-flows before interest and taxes will be, with equal probability, either 170.0% of its Time-t-1 after-tax operating cash-flows in State u, 80.0% of Time-t-1 its after-tax operating cash-flows in State m, or 50.0% of Time-t-1 after-tax operating cash-flows in State d. The market return, $k_{M,s}$, is 45.0% in State u, 20.0% in State m and -35.0% in State d. The risk-free rate of interest, k_F, is 5.0%. The firm's pre-tax operating cash flow is correlated with the return to the market. P_s is the Time-t-1 value of \$1 received in State s at Time t. The corporate tax rate, τ_C, is 40.0%.

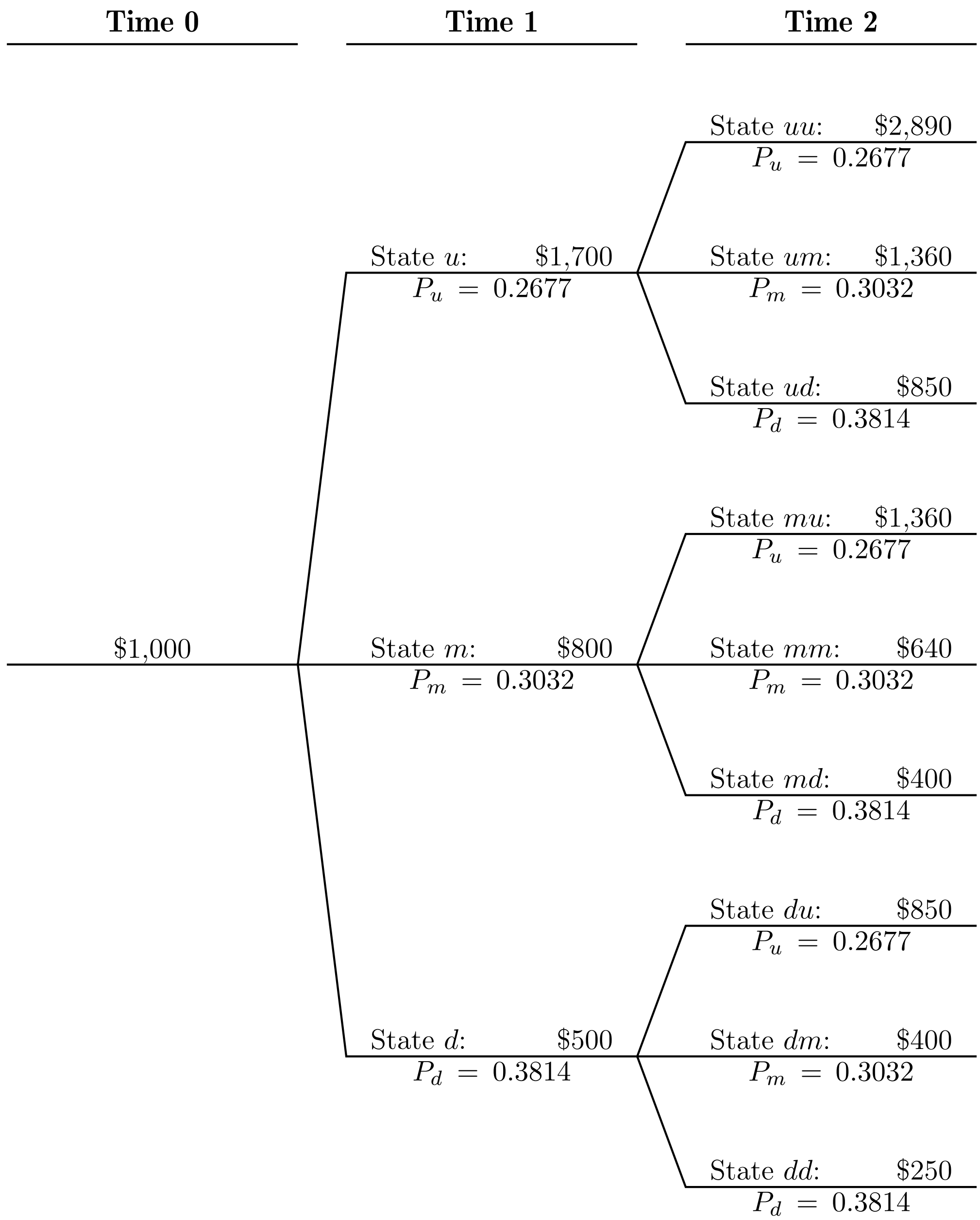

Table 1. Firm valuation under different contractual interest specifications

Miles and Ezzell (1980, 1985) and Ezzell and Miles (1983) assume the firm maintains a fixed leverage ratio into perpetuity and promises to pay interest at the end of each period proportional to its debt's beginning-of-period value. Berk and DeMarzo (2024), and implicitly Ruback (2002), assume the firm maintains a fixed leverage ratio into perpetuity and promises to pay interest at the end of each period proportional to its just-realized operating cash flow.

	Valuation element	Value	Notation
A.	**Common assumptions**		
	Risk-free rate	5.00%	k_F
	Return on operating assets	12.56%	k_A
	Tax rate	30.00%	τ_C
	Leverage ratio	40.00%	L
	Expected free cash flow	\$700.00	$\overline{FCF}$
B.	**Miles and Ezzell (1980, 1985) and Ezzell and Miles (1983) contractual interest**		
	Return on debt	5.00%	k_D
	Value of the unlevered firm	\$5,573.25	$V_0^U = \overline{FCF} \,/\, k_A$
	$WACC = k^*$	11.92%	$WACC = k^* = k_A - \tau_C k_D L \left(\frac{1+k_A}{1+k_D}\right)$
	Value of the levered firm	\$5,874.06	$V_0^L = \overline{FCF} \,/\, k^*$
	Return on equity	17.53%	$k_S = k_A + (k_A - k_D)\left(1 - \frac{\tau_C k_D}{1+k_D}\right)\left(\frac{L}{1-L}\right)$
	Value of tax-shield asset	\$300.81	$V_0^{TS} = \tau_C k_D L V_0^L \left(\frac{1}{1+k_D}\right)\left(\frac{1+k_A}{k_A}\right)$
	Expected debt	\$2,349.62	$\overline{D} = D_0 = L V_0^L$
	Expected interest tax shield	\$35.24	$\overline{TS} = \tau_C k_D D_0$
	Capital cash flow	\$735.24	$\overline{CCF} = \overline{FCF} + \overline{TS}$
	$WACA$ or pre-tax $WACC$	12.52%	$WACA = k_A - (k_A - k_D)\left(\frac{\tau_C k_D L}{1+k_D}\right)$
	Value of the levered firm	\$5,874.06	$V_0^L = \overline{CCF} \,/\, WACA$
C.	**Berk and DeMarzo (2024) contractual interest**		
	Return on debt	12.56%	$k_D = k_A$
	Payout ratio	45.45%	$\lambda = L \,/\, (1 - \tau_C L)$
	Value of the unlevered firm	\$5,573.25	$V_0^U = \overline{FCF} \,/\, k_A$
	$WACC = k^*$	11.05%	$WACC = k^* = k_A (1 - \tau_C L)$
	Value of the levered firm	\$6,333.24	$V_0^L = \overline{FCF} \,/\, k^*$
	Return on equity	12.56%	$k_S = k_A + (k_A - k_D)\left(1 - \frac{\tau_C k_D}{1+k_D}\right)\left(\frac{L}{1-L}\right)$
	Expected debt	\$2,533.29	$\overline{D} = D_0 = L V_0^L$
	Expected interest tax shield	\$95.45	$\overline{TS} = \tau_C k_D D_0$
	Value of tax-shield asset	\$759.99	$V_{TS} = TS \,/\, k_A$
	Capital cash flow	\$795.45	$\overline{CCF} = \overline{FCF} + \overline{TS}$
	$WACA$ or pre-tax $WACC$	12.56%	$WACA = k_A - (k_A - k_D)\left(\frac{\tau_C k_D L}{1+k_D}\right)$
	Value of the levered firm	\$6,333.24	$V_0^L = \overline{CCF} \,/\, WACA$

Table 2. The value of $WACC$, firm $WACA$, CCF and the levered at different leverage ratios, tax rates and returns on debt

The required rate of return on the unlevered firm's operating assets and equity, k_A, is 12.56%, the firm's perpetual pre-tax operating cash flows, $\overline{X}$, are \$1,000.00 per year, and the required rate of return on debt, k_D, is assumed to equal the required rate of return on the tax-shield asset, k_I. The firm's weighted-average cost of capital, $WACC$, and the weighted-average cost of assets, $WACA$, are computed for different tax rates, τ_C, and leverage ratios, L, using

$$WACC = k_A - \tau_C k_D L \left(\frac{1+k_A}{1+k_D}\right) \text{ and } WACA = k_A - (k_A - k_D)\left(\frac{\tau_C k_D L}{1+k_D}\right).$$

The firm's expected perpetual periodic capital cash flows are $\overline{CCF} = \overline{FCF} + \tau_C k_D L V_0^L$, where the firm's expected free cash flows are $\overline{FCF} = \overline{X}(1-\tau_C)$. The value of the levered firm is

$$V_0^L = \frac{\overline{FCF}}{WACC} = \frac{\overline{CCF}}{WACA}.$$

Leverage ratio (L)	Tax rate 0.0%	10.0%	30.0%	50.0%	70.0%	90.0%
A. $k_I = k_D = 5.0\%$[a]						
1. $WACC$						
0.00%	12.56%	12.56%	12.56%	12.56%	12.56%	12.56%
20.00%	12.56%	12.45%	12.24%	12.02%	11.81%	11.60%
40.00%	12.56%	12.35%	11.92%	11.49%	11.06%	10.63%
60.00%	12.56%	12.24%	11.60%	10.95%	10.31%	9.67%
80.00%	12.56%	12.13%	11.27%	10.42%	9.56%	8.70%
2. $WACA$						
0.00%	12.56%	12.56%	12.56%	12.56%	12.56%	12.56%
20.00%	12.56%	12.55%	12.54%	12.52%	12.51%	12.50%
40.00%	12.56%	12.55%	12.52%	12.49%	12.46%	12.43%
60.00%	12.56%	12.54%	12.50%	12.45%	12.41%	12.37%
80.00%	12.56%	12.53%	12.47%	12.42%	12.36%	12.30%
3. $\overline{CCF}$ $(= \overline{FCF}$ when $L = 0.00\%)$						
0.00%	\$1,000.00	\$900.00	\$700.00	\$500.00	\$300.00	\$100.00
20.00%	1,000.00	907.23	717.16	520.79	317.78	107.76
40.00%	1,000.00	914.58	735.24	543.52	337.98	116.93
60.00%	1,000.00	922.06	754.33	568.48	361.11	127.93
80.00%	1,000.00	929.68	774.51	596.01	387.88	141.38
4. V_0^L						
0.00%	\$7,961.78	\$7,165.61	\$5,573.25	\$3,980.89	\$2,388.54	\$796.18
20.00%	7,961.78	7,227.29	5,719.70	4,158.35	2,540.31	862.43
40.00%	7,961.78	7,290.05	5,874.06	4,352.37	2,712.67	940.70
60.00%	7,961.78	7,353.90	6,036.98	4,565.38	2,910.14	1,034.60
80.00%	7,961.78	7,418.89	6,209.20	4,800.31	3,138.60	1,149.32

[a] The debt is default-risk free at all leverage ratios.

Table 2. Continued

Leverage ratio (L)	Tax rate 0.0%	10.0%	30.0%	50.0%	70.0%	90.0%
B. $k_I = k_D = 10.0\%$						
1. $WACC$						
0.00%	12.56%	12.56%	12.56%	12.56%	12.56%	12.56%
20.00%	12.56%	12.36%	11.95%	11.54%	11.13%	10.72%
40.00%	12.56%	12.15%	11.33%	10.51%	9.69%	8.88%
60.00%	12.56%	11.95%	10.72%	9.49%	8.26%	7.03%
80.00%	12.56%	11.74%	10.10%	8.47%	6.83%	5.19%
2. $WACA$						
0.00%	12.56%	12.56%	12.56%	12.56%	12.56%	12.56%
20.00%	12.56%	12.56%	12.55%	12.54%	12.53%	12.52%
40.00%	12.56%	12.55%	12.53%	12.51%	12.49%	12.48%
60.00%	12.56%	12.55%	12.52%	12.49%	12.46%	12.43%
80.00%	12.56%	12.54%	12.50%	12.47%	12.43%	12.39%
3. $\overline{CCF}$ $\left(= \overline{FCF}\right.$ when $\left.L = 0.00\%\right)$						
0.00%	$1,000.00	$900.00	$700.00	$500.00	$300.00	$100.00
20.00%	1,000.00	914.45	734.32	541.58	335.56	115.52
40.00%	1,000.00	929.16	770.49	587.05	375.95	133.87
60.00%	1,000.00	944.12	808.67	636.96	422.23	155.87
90.00%	1,000.00	959.35	849.02	692.01	475.76	182.75
4. V_0^L						
0.00%	$7,961.78	$7,165.61	$5,573.25	$3,980.89	$2,388.54	$796.18
20.00%	7,961.78	7,284.30	5,859.68	4,333.98	2,696.04	933.00
40.00%	7,961.78	7,406.99	6,177.16	4,755.81	3,094.43	1,126.61
60.00%	7,961.78	7,533.88	6,531.00	5,268.60	3,630.97	1,421.60
80.00%	7,961.78	7,665.20	6,927.85	5,905.34	4,392.60	1,925.88